PARAMETER PLANE TECHNIQUES FOR ACTIVE FILTER SYSTEMS

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FOR ACTIVE FILTER SYSTEMS

by

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Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL
December 1967

ABSTRACT

Since the introduction of parameter plane techniques by Siljak in 1964, much effort has been devoted to the application of these techniques to various control system problems.

This paper concerns the application of parameter plane techniques to the design of active filter systems.

Constant bandwidth curves on the parameter plane are compared with frequency response curves to validate the constant bandwidth curves as accurate representations of active filter performance. This in turn implies that the constant bandwidth curves can be used in the design of active filter systems.

Also presented is an example of the use of constant zeta and omega curves on the parameter plane to manipulate the pole locations of individual circuits to achieve a desired overall system configuration with the proper frequency scaling.

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1. Introduction

The use of operational amplifiers in active filters increases the flexibility with which signal filters can be designed. Primary reasons for this increased design flexibility are the elimination of the need for inductors and of the need for impedance matching. Active filter circuits can be used over a wide frequency range, oftentimes with the added benefits of less cost, smaller size and less weight. [2] These features suggest that active filters provide a suitable answer to a wide variety of filtering problems, such as, aerospace medicine data collection and the analysis of vibrational fatigue in aircraft structures.

This paper is a case study based upon an active filter parallel-T filter designed by Mr. William Kerwin of NASA's Ames Laboratory and is primarily concerned with establishing that parameter plane techniques can be effectively applied to design of active filter systems. Figure 1 is a circuit diagram of the filter and Figure 2 illustrates the observed experimental filter performance of that circuit. The experimental performance will serve as the fundamental reference for the evaluation of the parameter plane design techniques used in this paper.

Two general types of parameter plane curves will be considered: (1) constant bandwidth curves and (2) constant zeta and omega curves.

Hollister developed equations for displaying curves of constant bandwidth on the parameter plane and defined a constant bandwidth curve as: [3]

> A constant bandwidth curve for $G(j\omega_b) = M$ is a curve drawn upon the parameter plane which specifies the relation between the parameters necessary if the transfer function $G(s)$, which is a function of the parameters, is to have magnitude M at the real frequency ω_b.

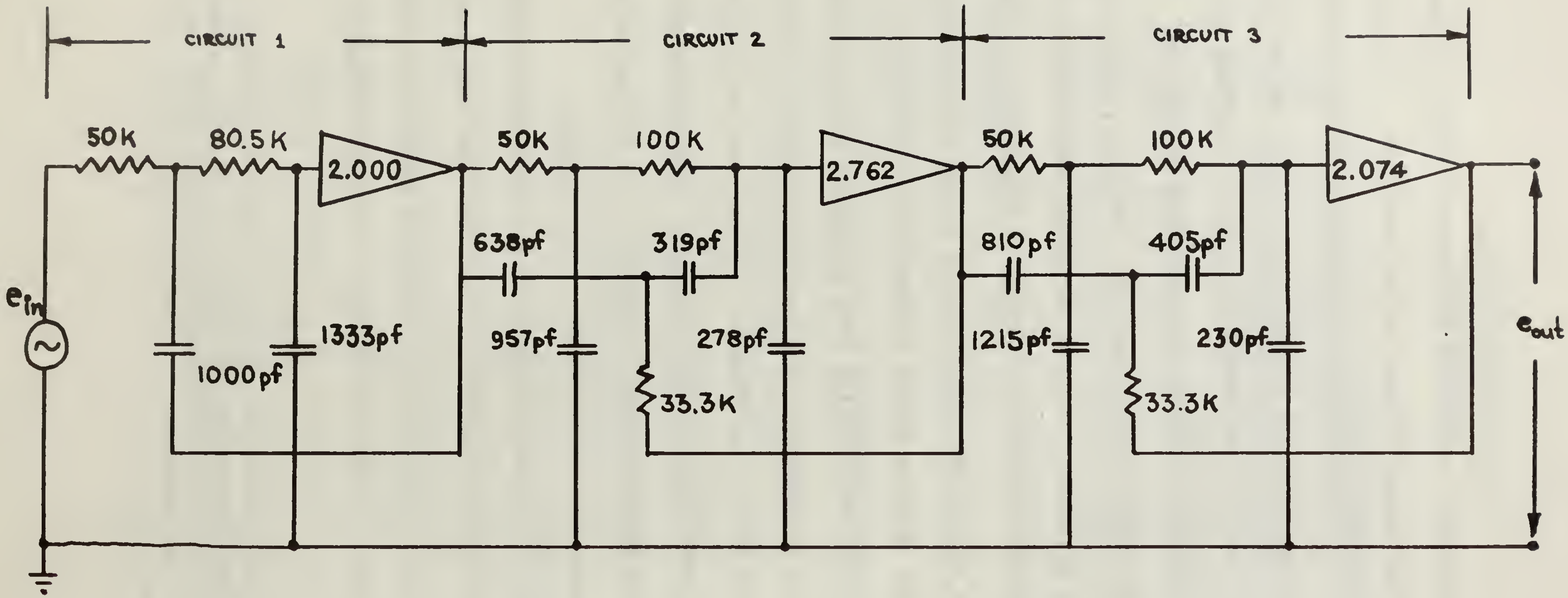

Figure 1. RC Elliptic Function Filter

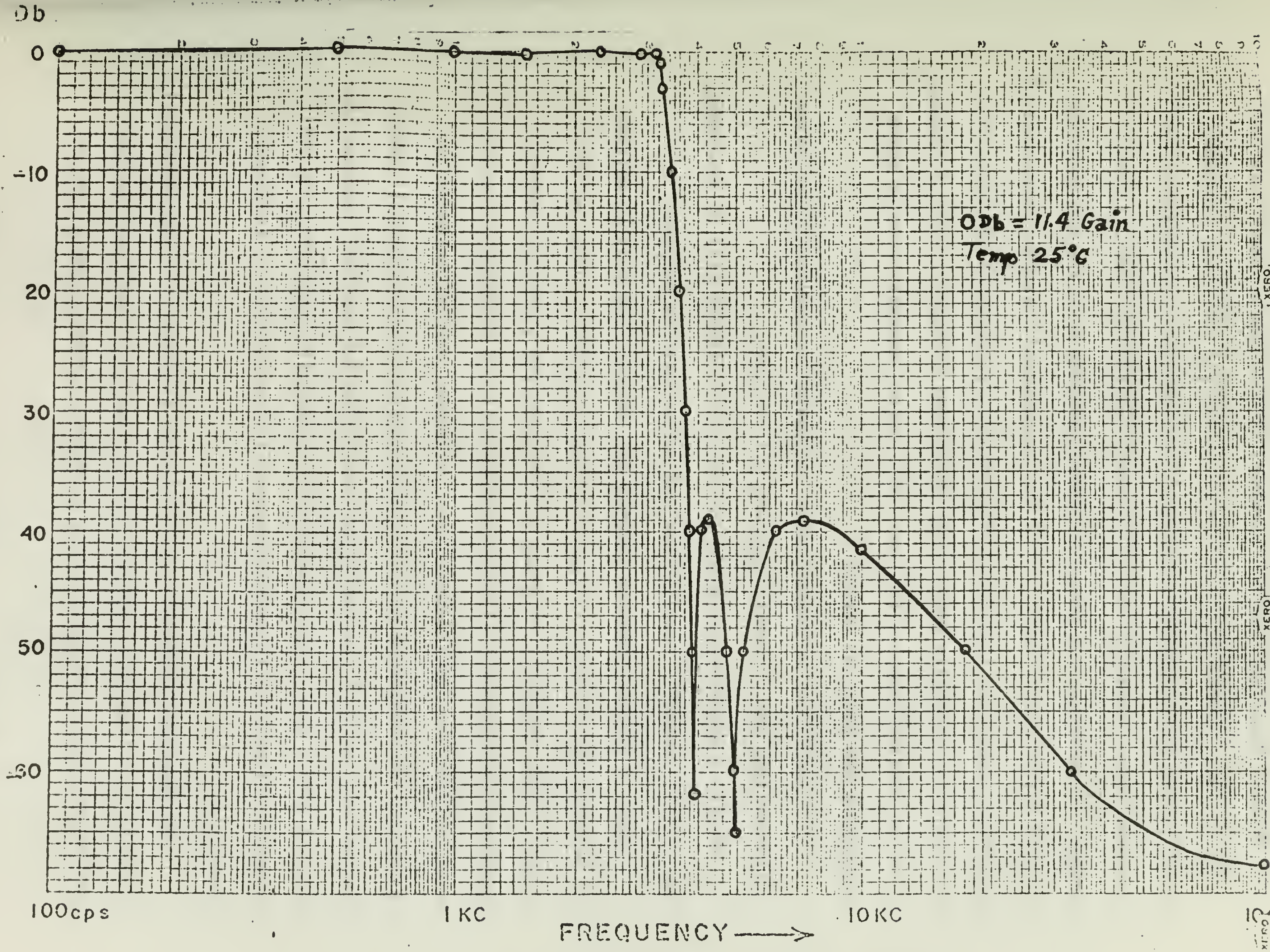

Figure 2. Experimental Filter Performance

Nutting developed computer programs to plot the constant zeta and constant omega curves on the parameter plane. [4] If these parameter plane curves are frequency normalized, then frequency scaling must be performed when cascading several circuits together to form a filter system. A technique for proper frequency scaling to allow the manipulation of system pole locations will be demonstrated. Proper conversion of parameter plane root locations into correct frequency scaled values for the individual circuits permits the individual circuit transfer functions to be multiplied together to obtain the system transfer function which represents the physical system.

In establishing parameter plane techniques as applicable to active filter design, the following approach will be made.

First, a mathematical model for the filter system will be developed, that is, the transfer function for the system will be derived.

Then, a digital computer program for the CDC 1604 will be written to evaluate the frequency response of the mathematical model of the system. This frequency response will be compared with the observed experimental response. If the performance of the mathematical model conforms with the actual experimental response then the mathematical model will be considered a valid representation of the physical system. Furthermore, each of the individual circuits which together comprise the overall system will be considered to be accurately represented by their respective individually derived mathematical models, i.e., transfer functions.

The frequency response of the transfer functions for the individual circuits will then be considered accurate enough to serve as standards for the evaluation of the constant bandwidth curves on the parameter plane. Good correlation between the constant bandwidth curves and the corresponding frequency response curves then implies that both of the curves accurately describe circuit performance and that the parameter plane can, therefore, be used as a design tool.

2. Mathematical Modeling of the System

Evaluation of the effectiveness of parameter plane techniques could be accomplished in several ways. One method would be the Hardware approach. Use parameter plane techniques to determine the values of selected parameters which will produce a given performance, then actually build the system and adjust the chosen parameters to the specified values. Observe the experimental performance and compare with the desired performance to substantiate the validity of the design technique.

A second method is simulation. In this case a mathematical model of the system is developed and the performance of the system is evaluated on either a digital or analog computer. When facilities are available for either analog or digital simulation these methods have the advantage of rapid evaluation and excellent flexibility.

Even if the first method of hardware analysis is applied, the development of the parameter plane curves requires an accurate mathematical model of the system.

Therefore the first step in design using the parameter plane is to develop the mathematical model of the system.

2.1. Derivation of Transfer Functions Through Signal Flow Graph Analysis

The mathematical modeling of the filter system shown in Figure 1 will be based upon the analysis techniques of the signal flow graph introduced by S. J. Mason. [5] The system will be divided into the three circuits indicated in Figure 1. Transfer functions for the individual circuits will be derived using signal flow graph analysis. The overall system transfer function will be the product of the individual circuit transfer functions. This overall system transfer function will be evaluated by a computer program to be presented in Section 3.

2.1.1. Signal Flow Graph Analysis of Circuit 1.

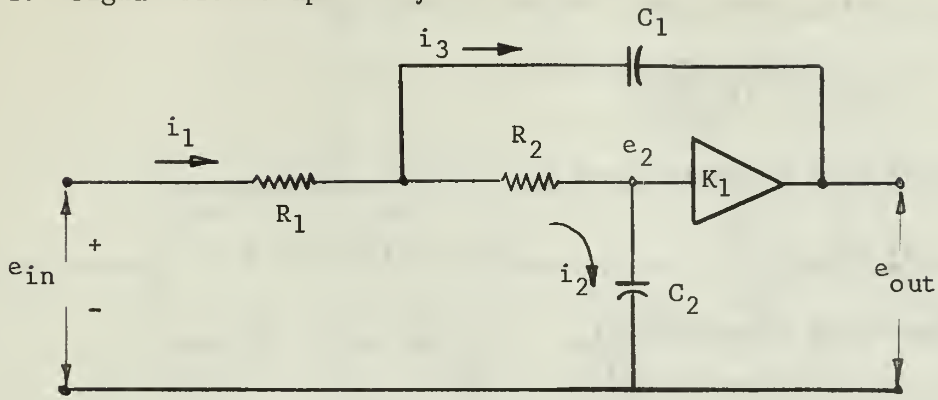

Figure 3. Circuit Diagram for Circuit 1

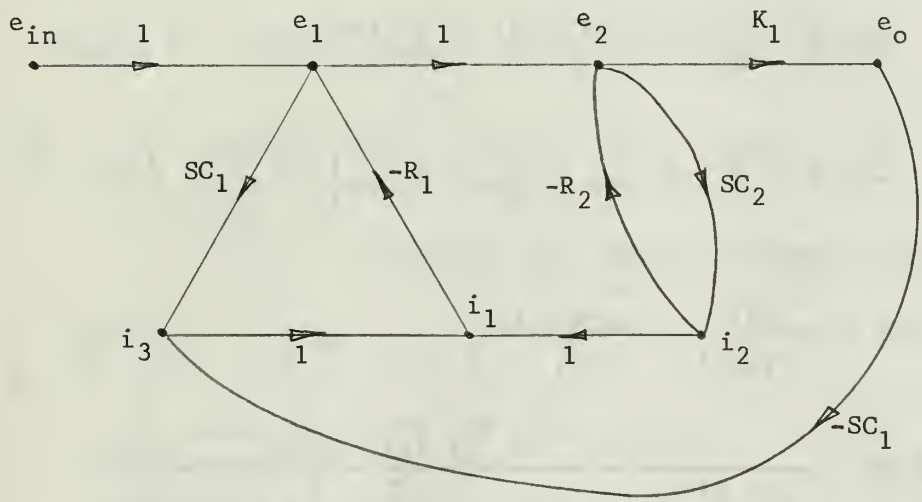

Figure 4. Signal Flow Graph for Circuit 1

The loops of the signal-flow graph, $L_K^{(1)}$

$L_1 = -R_1C_1S = -\tau_{11}S$ Nodes: (e_1, i_3, i_1)

$L_2 = -R_1C_2S = -\tau_{12}S$ (e_1, e_2, i_2, i_1)

$L_3 = -R_2C_2S = -\tau_{22}S$ (e_2, i_2)

$L_4 = K_1R_1C_1S = K\tau_{11}S$ $(e_2, e_o, i_3, i_1, e_1)$

where

$$\tau_{ij} \triangleq R_iC_j$$

Non-touching loops taken two at a time, $L_k^{(2)}$

$$L_1 L_3 = \tau_{11} \; \tau_{22} S^2$$

Forward path transmissions, p_k

$$p_1 = K \qquad\qquad (e_{in}, e_1, e_2, e_o)$$

Forward path cofactors, Δ_k

$$\Delta_1 = 1$$

Graph determinant, Δ

$$\Delta = 1 - \sum_k L_k^{(1)} + \sum_k L_k^{(2)} - \sum L_k^{(3)} + \ldots.$$

$$\Delta = 1 + \left[\tau_{11} + \tau_{12} + \tau_{22} - K\tau_{11}\right] S + \tau_{11} \; \tau_{22} S^2$$

Transfer function, $T_1(S)$, for circuit 1

$$T(S) = \frac{E(S)}{E_{in}(S)} \quad \frac{\sum p_k \Delta_k}{\Delta}$$

$$T_1(S) = \frac{K_1/\tau_{11}\tau_{22}}{S^2 + \left[\frac{1-K_1}{\tau_{22}} + \frac{1}{\tau_{11}} + \frac{\tau_{12}}{\tau_{11}\tau_{22}}\right] S + \frac{1}{\tau_{11}\tau_{22}}}$$

$$\tau_{11} = R_1 C_1$$

$$\tau_{12} = R_1 C_2 \qquad\qquad \frac{\tau_{12}}{\tau_{11}\tau_{22}} = \frac{1}{\tau_{21}} \triangleq \frac{1}{R_2 C_1}$$

$$\tau_{22} = R_2 C_2$$

2.1.2 Signal-flow Graph Analysis of Active Parallel-T with Resistive Feedback.

The circuit diagram shown below is an active parallel-T network and represents both circuit 2 and circuit 3.

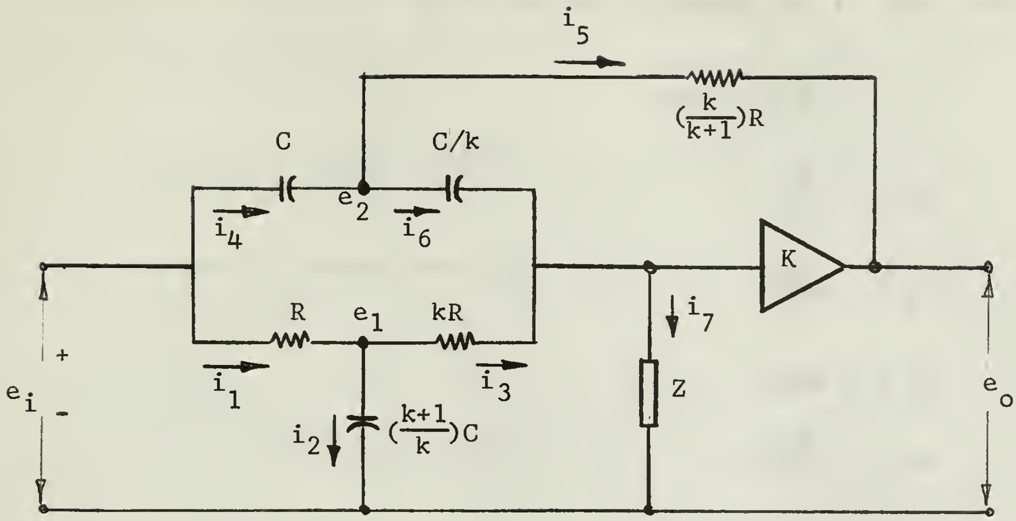

Figure 5. Circuit Diagram, Active Parallel-T Network

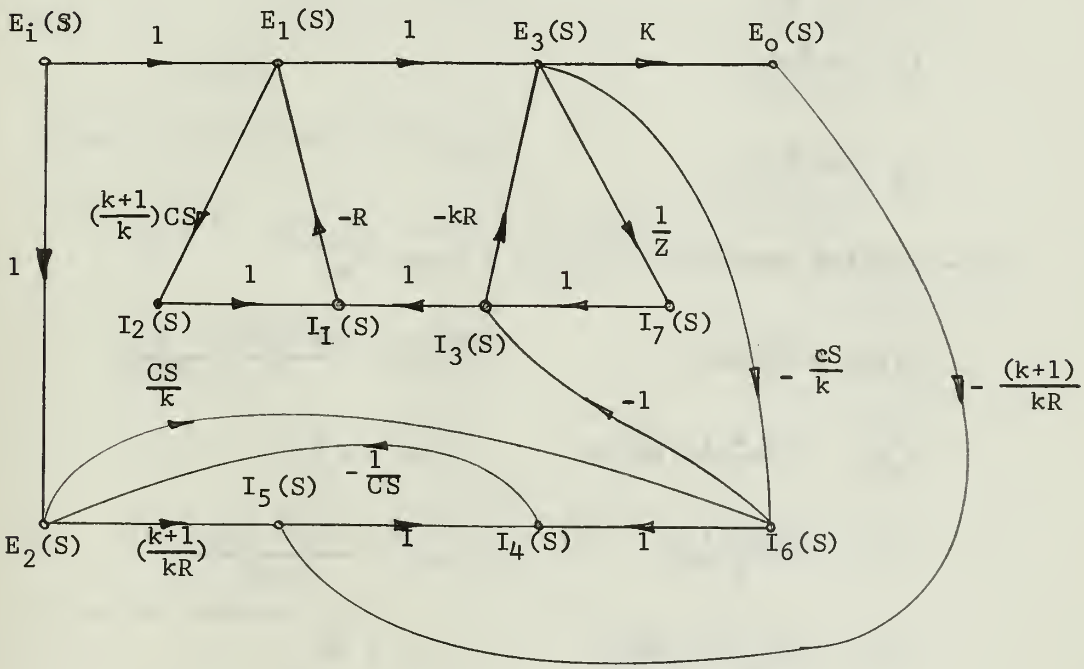

Figure 6. Signal Flow Graph, Active Parallel-T Network

The loops of the signal-flow graph, $L_k^{(1)}$

$$L_1 = -\left(\frac{k+1}{k}\right)RCS$$

$$L_2 = -\frac{R}{Z}$$

$$L_3 = -\frac{kR}{Z}$$

$$L_4 = -RCS$$

$$L_5 = -\frac{RCS}{k}$$

$$L_6 = -\frac{k+1}{k}\left(\frac{1}{RCS}\right)$$

$$L_7 = -\frac{1}{k}$$

$$L_8 = K\frac{k+1}{k^2}$$

$$L_9 = K\left(\frac{k+1}{k}\right)$$

Non-touching loops taken two at a time, $L_k^{(2)}$

$$L_1L_3 = R\frac{k+1}{Z}RCS$$

$$L_1L_4 = \left(\frac{k+1}{k}\right)(RCS)^2$$

$$L_1L_6 = \left(\frac{k+1}{k}\right)^2$$

$$L_1L_7 = \frac{k+1}{k}\,\frac{RCS}{k}$$

$$L_1L_9 = -\left(\frac{k+1}{k}\right)^2 KRCS$$

$$L_2L_6 = \left(\frac{k+1}{k}\right)\frac{R}{ZRCS}$$

$$L_2L_7 = \frac{R}{kZ}$$

$$L_3L_6 = \frac{(k+1)R}{ZRCS}$$

$$L_3L_7 = \frac{R}{Z}$$

$$L_4L_6 = \frac{k+1}{k}$$

$$L_5L_6 = \frac{k+1}{k^2}$$

Non-touching loops taken three at a time, $L_k^{(3)}$

$$L_1L_3L_6 = -\frac{(k+1)^2}{kZ}R \qquad L_1L_4L_6 = -\left(\frac{k+1}{k}\right)^2 RCS$$

$$L_1L_3L_7 = -\left(\frac{k+1}{k}\right)\frac{RCS}{Z}$$

Forward path transmissions, P_k

$$p_1 = K$$

$$p_2 = (1)\left(\frac{CS}{k}\right)(-1)(+1)(-R)(1)K$$

$$= \frac{K}{k}RCS$$

$$p_3 = (1)\left(\frac{CS}{k}\right)(-1)(-kR)K$$

$$= KRCS$$

Forward path cofactors, Δ_k

$$\Delta_1 = 1 - (L_6 + L_7) = 1 + \left(\frac{k+1}{k}\right)\frac{1}{RCS} + \frac{1}{k}$$

$$= \frac{k+1}{k}\left[1 + \frac{1}{RCS}\right]$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - L_1 = 1 + \left(\frac{k+1}{k}\right)RCS$$

Graph determinant, Δ

$$\Delta = 1 - \sum L_k^{(1)} + \sum L_k^{(2)} - \sum L_k^{(3)} + \ldots\ldots$$

$$-\sum_{k=1}^{9} L^{(1)} = \left(\frac{k+1}{k} + 1 + \frac{1}{k}\right)RCS + (k+1)\frac{R}{Z} -$$

$$K\left(\frac{k+1}{k}\right)\left(1 + \frac{1}{K}\right) + \frac{1}{k} + \left(\frac{k+1}{k}\right)\frac{1}{RCS}$$

$$+\sum L_k^{(2)} = \left(\frac{k+1}{k}\right)(RCS)^2 + \left\{\frac{k+1}{Z}R + \frac{k+1}{k^2} - \left(\frac{k+1}{k}\right)^2 K\right\}$$

$$RCS + \left(\frac{k+1}{k}\right)\left\{\left(\frac{k+1}{k}\right) + 1 + \frac{1}{k}\right\} + \frac{R}{Z}\left(1 + \frac{1}{k}\right) +$$

$$\frac{R}{Z}\left(1 + \frac{1}{k}\right) + \left(\frac{k+1}{k}\right)\frac{R}{Z}\left\{1 + \frac{1}{k}\right\}\frac{1}{RCS}$$

$$-\sum L_k^{(3)} = \left(\frac{k+1}{k}\right)\left[\left(\frac{k+1}{k} + \frac{1}{Z}\right)RCS + R\frac{k+1}{Z}\right]$$

Let RCS = p

$$\Delta = \frac{k+1}{k}p^2 + \frac{k+1}{k}\left[2 + \frac{k}{Z} + \frac{1}{k} - \frac{k+1}{k}K + \right.$$

$$\left.\frac{k+1}{k} + \frac{1}{Z}\right]p + 1 + \frac{1}{k} + \frac{k+1}{k}\left(\frac{R}{Z}\right)(k+1) +$$

$$\left(\frac{k+1}{k}\right)^2(2 - K) + \frac{(k+1)^2}{kZ}R + \left(\frac{k+1}{k}\right)$$

$$\left[1 + \frac{k+1}{k}\frac{R}{Z}\right]\frac{1}{p}$$

$$\Delta = \frac{k+1}{k}\left[p^2 + \left\{\frac{3k+2}{k} + \frac{k+1}{Z}R - K\frac{k+1}{k}\right\}p + \right.$$

$$\left\{\frac{3k+2}{k} + \frac{2(k+1)R}{Z} - K\left(\frac{k+1}{k}\right)\right\} +$$

$$\left.\left\{1 + \frac{k+1}{k}\frac{R}{Z}\right\}\frac{1}{p}\right]$$

Transfer function

$$T(S) = \frac{\sum P_k \Delta_k}{\Delta}$$

$$T(S) = \frac{K\left(\frac{k+1}{k}\right)\left[p^2 + p + 1 + \frac{1}{p}\right]}{\left(\frac{k+1}{k}\right)\left[p^2 + \left\{\frac{3k+2}{k} + \frac{k+1}{Z}R - \frac{k+1}{k}K\right\}p + \left\{\frac{3k+2}{k} + \frac{2(k+1)R}{Z} - \frac{k+1}{k}K\right\} + \left\{1 + \frac{(k+1)}{k}\frac{R}{Z}\right\}\frac{1}{p}\right]}$$

multiply numerator and denominator by $\frac{kp}{k + 1}$, then factor (p + 1) from numerator and denominator

$$T(S) = \frac{K\left[p^2 + 1\right]}{p^2 + \left(\frac{k + 1}{k}\right)\left(2 + \frac{kR}{Z} - K\right)p + 1 + \frac{k + 1}{Z} R}$$

where

$$p = RCS$$

Consideration of load impedance, Z, let

$$Z = \frac{1}{C_Z S}$$

then

$$T(S) = \frac{K\left[p^2 + 1\right]}{p^2 + \frac{k + 1}{k}(2 - K)p + 1 + (k + 1)RC_Z Sp + (k + 1)RC_Z S} \qquad (2.2\text{-}1)$$

then define

$$C' \triangleq \frac{C_Z}{C}$$

$$C_Z = C'C$$

$$RC_Z S = RC'CS = C'p$$

Substitute into equation (2.2-1)

$$T(S) = \frac{K\left[p^2 + 1\right]}{\left[(k + 1)C' + 1\right]p^2 + \left[\frac{k + 1}{k}(2 - K) + (k + 1)C'\right]p + 1}$$

where

$$p = RCS$$

$$C' = \frac{C_Z}{C}$$

2.2. Frequency Scaling the Transfer Function

Whenever several circuits are cascaded it becomes necessary to frequency scale the complete network to some reference frequency f_o

prior to evaluating the overall frequency response.

In the RC elliptical function filters under consideration the "notch frequency" is

$$f_n = \frac{1}{2\pi\omega_n} = \frac{RC}{2\pi}$$

Define

$$a = \frac{f_n}{f_o} = \frac{\omega_n}{\omega_o}$$

then

$$\omega_n = \frac{1}{RC} = a\omega_o \triangleq \frac{a}{R_oC_o}$$

$$\frac{1}{RCS} = \frac{a}{R_oC_oS} = \frac{1}{p} = \frac{a}{p_o} \longrightarrow p = \frac{p_o}{a}$$

The normalized transfer function then becomes

$$T_o(S) = \frac{K\left[(\frac{p_o}{a})^2 + 1\right]}{\left[(k+1)C'+1\right](\frac{p_o}{a})^2 + \left[\frac{k+1}{k}(2-K) + (k+1)C'\right]\frac{p_o}{a} + 1}$$

Multiply numerator and denominator times a^2

$$T_o(S) = \frac{K\left[p_o^2 + a^2\right]}{\left[(k+1)C' + 1\right]p_o^2 + a\left[\frac{k+1}{k}(2 - K) + (k + 1)C'\right]p_o + a^2}$$

Frequency scaling the transfer function:

For example, consider filter section 2

$R = 50\text{ K}$

$C = 638\text{ pf}$ $\qquad \omega_n = \frac{1}{R_2C_2} = 3.135 \times 10^4$

$k = 2.0$

$C' = \frac{278\text{ pf}}{638\text{ pf}} = 0.436$ $\qquad \omega_o \triangleq 2.0 \times 10^4$

$K = 2.762$ $\qquad a = \frac{\omega_n}{\omega_o} = 1.5675$

$$T_{02}(S) = \frac{2.762\left[p_o^2 + 2.4491\right]}{\left[(3x0.436) + 1\right]p_o^2 + 1.5649\left[\frac{3}{2}(-0.762) + (3x0.436)\right]p_o + 2.4491}$$

$$= \frac{2.762\left[p_o^2 + 2.4491\right]}{2.308p_o^2 + 1.5649(.165)p_o + 2.4491}$$

$$= \frac{\frac{2.762}{2.308}\ p_o^2 + 2.4491}{p_o^2 + 0.112p_o + 1.0611}$$

$$= \frac{1.1967\left[p_o^2 + 2.4491\right]}{p_o^2 + 0.112\ p_o + 1.0611}$$

Frequency scaling the transfer function.

Similarly for filter section 3

$$\left.\begin{array}{l} R = 50\ K \\ C = 810\ pf \end{array}\right\} \omega_n = \frac{1}{R_3C_3} = 2.469 \times 10^4 \text{ rad/sec}$$

$k = 2$

$$C' = \frac{230}{810} = 0.284$$

$K = 2.074$

$$(a)^2 = \left(\frac{\omega_n}{\omega_o}\right)^2 = 1.5220 \qquad a = 1.2336$$

$$T_{03}(S) = \frac{2.074\left[p_o^2 + 1.5220\right]}{\left[(3)(0.2840) + 1\right]p_o^2 + 1.234\left[(\frac{3}{2}(-0.074) + 08520\right]p_o + 1.5220}$$

$$= \frac{\frac{2.074}{1.8520}\left[p_o^2 + 1.5220\right]}{p_o^2 + 0.492p_o + 0.822}$$

$$= \frac{1.1198\left[p_o^2 + 1.5220\right]}{p_o^2 + 0.493p_o + 0.822}$$

Frequency scaling the transfer function.

For filter section 1

$$T(S) = \frac{K_1}{\tau_{11}\tau_{22}S^2 + \tau_{11}\tau_{22}\left[\frac{1-K_1}{\tau_{22}} + \frac{1}{\tau_{11}} + \frac{1}{\tau_{21}}\right]S + 1}$$

For this section the scaling factor, a, is defined:

$$a_o^2 = \frac{\tau_o^2}{\tau_{11}\tau_{22}} \qquad \therefore \quad \tau_{11}\tau_{22} = \frac{\tau_o^2}{a_o^2}$$

$$\tau_{11}\tau_{22}S^2 = \frac{\tau_o^2 S^2}{a_o^2} \triangleq \frac{p_o^2}{a_o^2}$$

$$T_o(s) = \frac{K_1}{\frac{p_o^2}{a_o^2} + \frac{\tau_o^2}{a_o^2}\left[\frac{1-K_1}{\tau_{22}} - \frac{1}{\tau_{11}} + \frac{1}{\tau_{21}}\right]S + 1}$$

$$= \frac{K_1}{\frac{p_o^2}{a_o^2} + \frac{\tau_o}{a_o^2}\left[\frac{1-K_1}{\tau_{22}} + \frac{1}{\tau_{11}} + \frac{1}{\tau_{21}}\right]p_o + 1}$$

$$\tau_{o1}(S) = \frac{a_o^2 K_1}{p_o^2 + \tau_o\left[\frac{1-K_1}{\tau_{22}} + \frac{1}{\tau_{11}} + \frac{1}{\tau_{21}}\right]p_o + a_o^2}$$

$$\tau_o = R_oC_o = \frac{1}{2 \times 10^4} = 0.50 \times 10^{-4}$$

$$\tau_{11} = R_1C_1 = (50 \times 10^3)(10^3 \times 10^{-12}) = 0.50 \times 10^{-4}$$

$$\tau_{22} = R_2C_2 = (80.5 \times 10^3)(1.333 \times 10^{-9}) = 1.0733 \times 10^{-4}$$

$$\tau_{21} = R_2C_1 = (80.5 \times 10^3)(10^{-9}) = 0.805 \times 10^{-4}$$

$$a_o^2 = \frac{\tau_o^2}{\tau_{11}\tau_{22}} = \frac{(0.5 \times 10^{-4})(0.5 \times 10^{-4})}{(0.5 \times 10^{-4})(1.0733 \times 10^{-4})} = \frac{1}{2.1466} = 0.4658$$

$$a_o = 0.6825$$

$$T_o(p_o) = \frac{(0.4658)(2.000)}{p_o^2 + (0.50\text{x}10^{-4})\left[\frac{1-2.000}{1.0733\text{x}10^{-4}} + \frac{1}{0.50\text{x}10^{-4}} + \frac{1}{0.805\text{x}10^{-4}}\right]S + 0.4658}$$

$$= \frac{(0.4658)(2.000)}{p_o^2 + (\frac{-0.500}{1.0733} + 1 + \frac{0.500}{0.805})p_o + 0.4658}$$

$$= \frac{(0.4658)(2.000)}{p_o^2 + (1 + 0.6211 - 04658)p_o + 0.4658}$$

Frequency normalized transfer function for section 1

$$T_{o1}(p_o) = \frac{0.9316}{p_o^2 + 1.1553p_o + 0.4658}$$

where

$$p_o = R_oC_oS$$

$$= 0.5 \times 10^{-4}S$$

3. Evaluation of the Mathematical Model

Does the mathematical model developed in Section 2 accurately represent the physical system? Can those transfer functions be used in developing the parameter plane curves?

To answer these questions the digital computer program, Program SYS RESP, written in FORTRAN 60, was developed to evaluate system frequency response when the system consists of several active circuits in cascade.

```
..JOB0600F, NAKAGAWA, G.R.
      PROGRAM SYS RESP
C     THIS PROGRAM MAY BE USED TO EVALUATE THE SYSTEM FREQUENCY
C     RESPONSE IN BOTH MAGNITUDE RATIO AND IN DB FOR A SYSTEM
C     CONSISTING OF UP TO FIVE ACTIVE STAGES WHEN THE NUMERATOR
C     POLYNOMIAL FOR THE TRANSFER FUNCTION OF EACH STAGE CAN BE
C     REPRESENTED BY
C
C                   XK*(B(1) + B(2)*S + B(3)*S**2)
C
C     AND THE DENOMINATOR POLYNOMIAL BY
C
C                   A(1) + A(2)*S + A(3)*S**2 + A(4)*S**3
C
C     THE FOLLOWING DATA CARDS WILL BE USED
C     CARD 1    FIRST LINE OF GRAPH TITLE           FORMAT(6A8)
C     CARD 2    SECOND LINE OF GRAPH TITLE          FORMAT(6A8)
C     CARD 3    NSTAGE,NUMPTS                       FORMAT(2I4)
C     CARD 4    W(0), INITIAL VALUE OF OMEGA        FORMAT(F10.4)
C     CARD 5    XK(1),GAIN COEFFICIENT,STAGE 1      FORMAT(F10.4)
C     CARD 6    B(1,J),NUMERATOR COEFFICIENTS,STAGE 1,
C                   IN ASCENDING ORDERS OF S'       FORMAT(4F10.4)
C     CARD 7    A(1,J),DENOMINATOR COEFFICIENTS,STAGE 1,
C                   IN ASCENDING ORDERS OF S        FORMAT(4F10.4)
C     CARD 8,9,10    XK(2), B(2,J) AND A(2,J) RESPECTIVELY
C     REPEAT THIS SEQUENCE TO ACCOMODATE THE REQUIRED NUMBER
C     OF STAGES
C
      DIMENSION K(5),B(5,5),A(5,5), ITITLE(12), W(400),
     1 BREAL(5,400),BIMAG(5,400), BMAG(5,400), DB(5,400),
     2AIMAG(5,400), AMAG(5,400),XK(5),ABS(900),AREAL(5,400),
     3DBSYS(900)
      DBSYS(0)= 0.000
      DBSYS (1) = 0.000
      W(0)= 0.000
      W(1) = 0.000
      LABEL = 4H
      READ 200, (ITITLE(I),I=1,6)
      READ 200, (ITITLE(I), I=7,12)
      READ 30, NSTAGE,NUMPTS
      READ 50, W(0)
      DO 60 J = 1, NUMPTS
      ABS(J) = 1.0
   60 CONTINUE
      DO 13 I = 1,NSTAGE
      READ  0, XK(I)
      READ 100, (B(I,K), K=1,4)
      READ 100, (A(I,K), K= 1,4)
      PRINT 130,XK(I),(B(I,K),K=1,3)
```

```
      PRINT 150,(A(I,K),K=1,4)
      DO 25 J = 1,NUMPTS
      W(J) = W(J-1) + 0.01
      BREAL(I,J) = B(I,1) - B(I,3)  *W(J)**2
      BIMAG(I,J) = B(I,2)*W(J)
      BMAG(I,J) = SQRTF(BREAL(I,J)**2 + BIMAG(I,J)**2)
      AREAL (I,J) = A(I,1) - A(I,3)* W(J)**2
      AIMAG(I,J) = A(I,2)* W(J) - A(I,4) * W(J)**3
      AMAG (I,J) = SQRTF (AREAL (I,J)**2 + AIMAG(I,J)**2 )
      TEMP = XK(I)*BMAG(I,J)/AMAG(I,J)
      ABS(J) = ABS(J)*TEMP
      DB(I,J) = 20.*LOG10F(TEMP)
   25 DBSYS(J) = DBSYS(J) + DB(I,J)
   13 CONTINUE
      DO 27 J = 50,150,5
      PRINT 310,(DB(I,J),I=1,3)
      PRINT 320, DBSYS(J)
   27 CONTINUE
      CALL DRAW(NUMPTS,W,DBSYS,0,0,LABEL,ITITLE,0,10.,0,1,0,2,8,
     1 12,1,LAST)
      CALL DRAW(NUMPTS,W,ABS,0,0,LABEL,ITITLE,0,0,0,1,0,2,8,
     1 12,1,LAST)
   30 FORMAT (2I4)
   50 FORMAT (F10.4)
  100 FORMAT (4F10.4)
  130 FORMAT (1X,30HTHE XK AND B COEFFICIENTS ARE ,/,4F10.4)
  150 FORMAT (1X,30HTHE A COEFFICIENTS ARE        ,/,4F10.4)
  200 FORMAT (6A8)
  310 FORMAT (5X,18HDB OF THE STAGES  ,3E18.6)
  320 FORMAT (5X,18HDBSYS EQUALS      ,E18.6)
      END
      END

JOB 0600 NAKAGAWA   PARALLEL-T ACTIVE FILTER
    X = W(J)      Y = DB OUT     SK = 2.0
  3 400
  .00
   .9316
  1.000
   .4658   1.1553   1.000
 1.1967
 2.4491   0.000    1.000
 1.0611   0.112    1.000
  1.1198
  1.5220  0.000    1.000
   .822   0.493    1.000
```

The system frequency response plotted by Program SYS RESP when the previously derived, frequency scaled circuit transfer functions are inserted as data is shown in Figure 7. Since it can be shown that 3.18 kc corresponds to a normalized omega equal to unity, comparison of this normalized system response with the experimental filter performance recorded in Figure 1 clearly indicates that the mathematical model is valid, that is, the transfer function is correct. In addition these results also indicate that the program will accurately simulate the performance of the physical system.

Validity of the system mathematical model implies:

(1) that each of the circuits which together comprise the system evaluated above can in turn be represented by its respective transfer function.

(2) that the performance of these individual circuit transfer functions can now be used in turn to evaluate the constant bandwidth curves on the parameter plane.

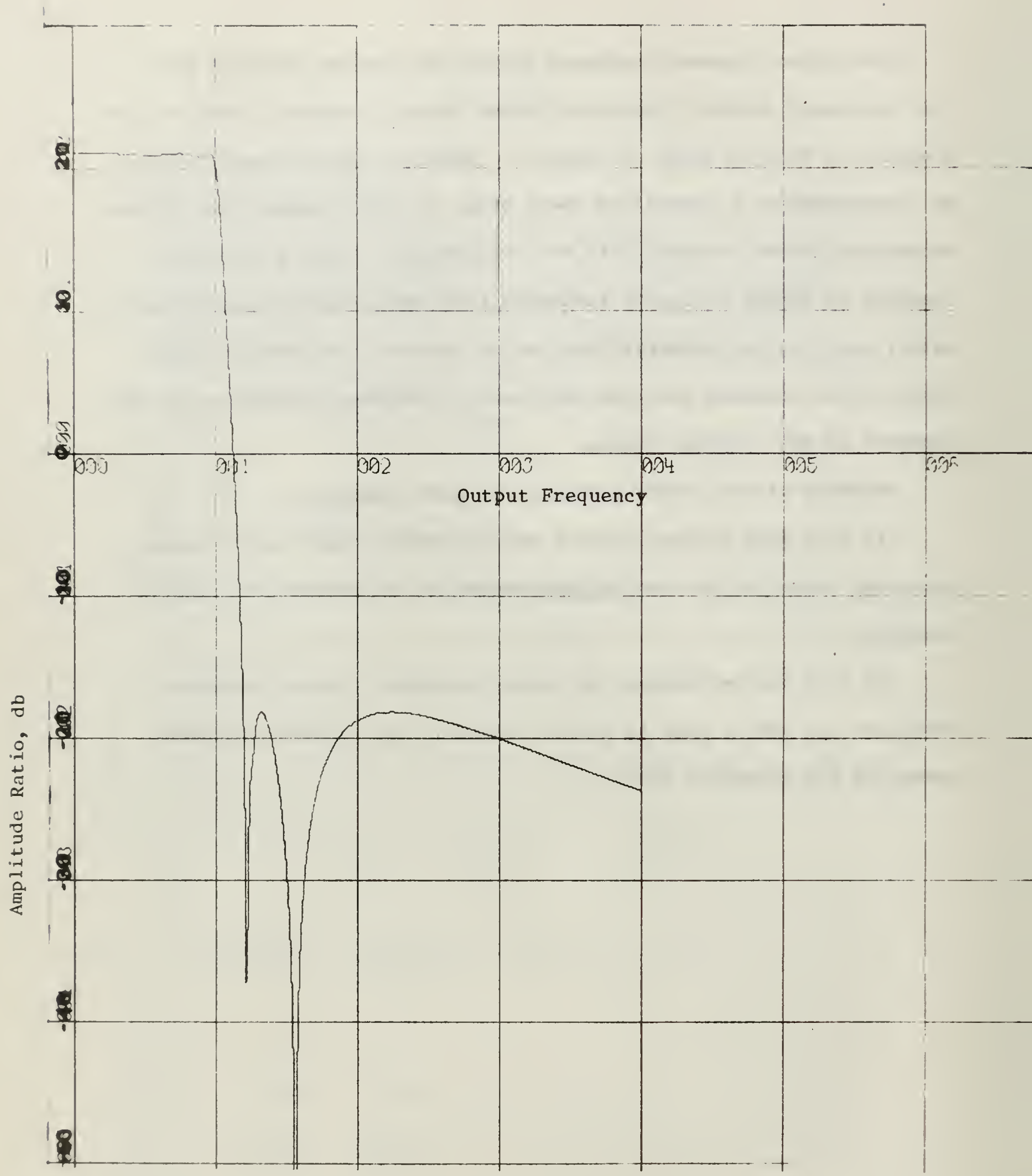

Figure 7. Digital Simulation of System Frequency Response

4. Active Filter Constant Bandwidth Curves on the Parameter Plane

The computer program included in Appendix A was written in FORTRAN 63 by Jean Bow, Programmer, Computer Facility, Naval Postgraduate School and modified slightly by the author to display constant bandwidth curves on the parameter plane. The program is based upon the constant bandwidth equations developed by Hollister and utilizes the transfer function for the active parallel-T network derived in Section 2.1.2.

The constant bandwitdh curves with amplitude contours in decibels are shown on the parameter plane plots of Figure 8-1 through Figure 8-18. As noted earlier each parameter plane plot of the constant bandwidth curves pertains to a fixed value of ω_b.

To obtain a sketch of the frequency response of the circuit, select the values of the two parameters on the abscissa and ordinate. This establishes an operating point on the parameter plane. Enter each plot at the selected operating point to determine magnitude, M, at the fixed value of ω_b represented by that plot. The frequency response curve can now be constructed by plotting M as a function of ω_b.

Is this technique valid, that is, is it accurate enough to be considered as a possible active filter design technique?

The computer program included in Appendix B was developed specifically to answer these questions. Written in FORTRAN 60, Program FREQRESP evaluates the frequency response of the parallel-T active filter.

Figure 9-1 is the graphical output for this program with SK, the ratio of the circuit element values, set at 2.0 and C, the normalized capacitance, equal to 0.20. The amplifier gain is varied from 1.6 to 5.4 to produce the family of curves shown, i.e., each curve represents

The following information applies to Figures 8-1 through 8-18:

X Variable	"Amplifier Gain, K"
X Scale	1 inch = 0.5
Y Variable	"Normalized Capacitance, C"
Y Scale	1 inch = 0.1

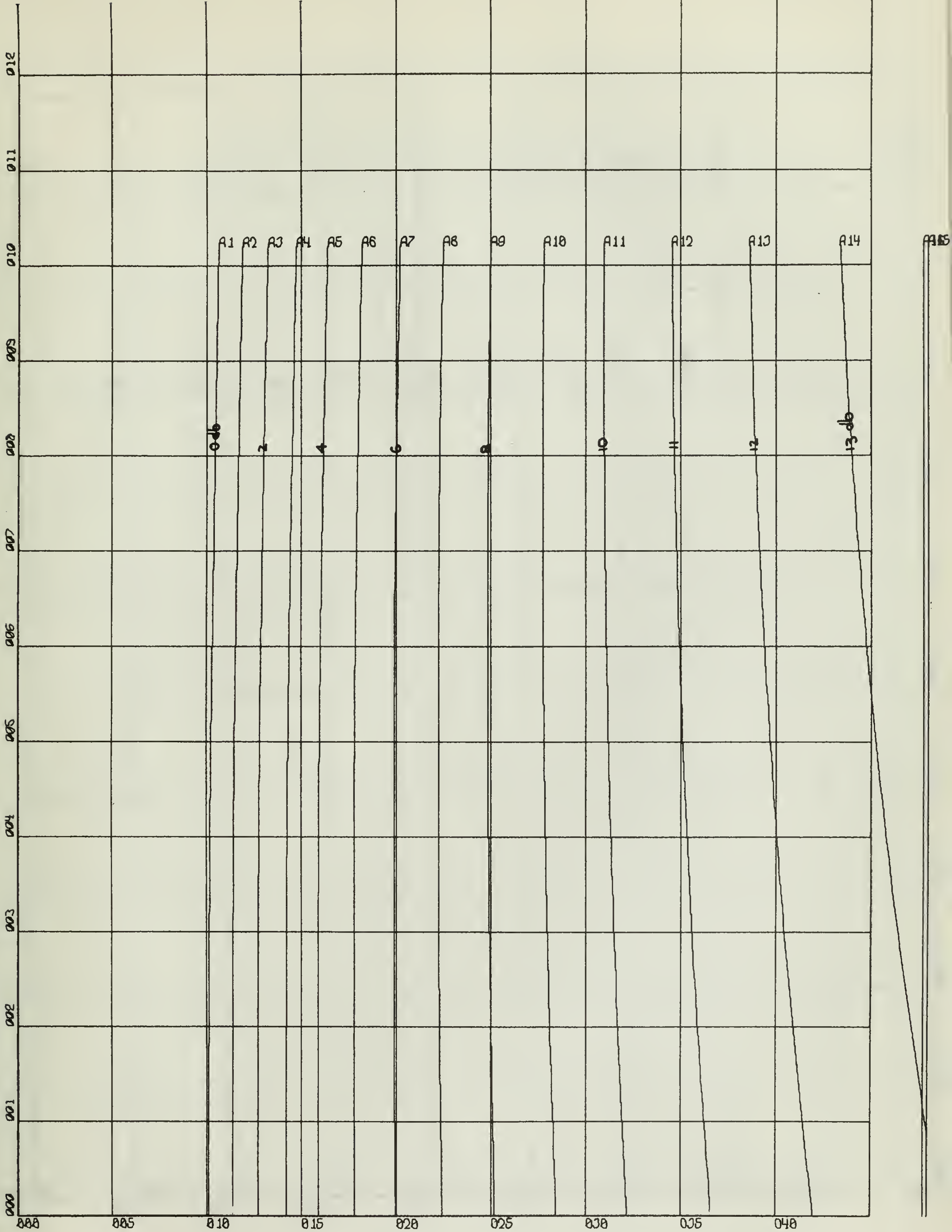

Figure 8-1. Active Filter Constant Bandwidth Curves on Parameter Plane k = 2.0 ω = 0.10

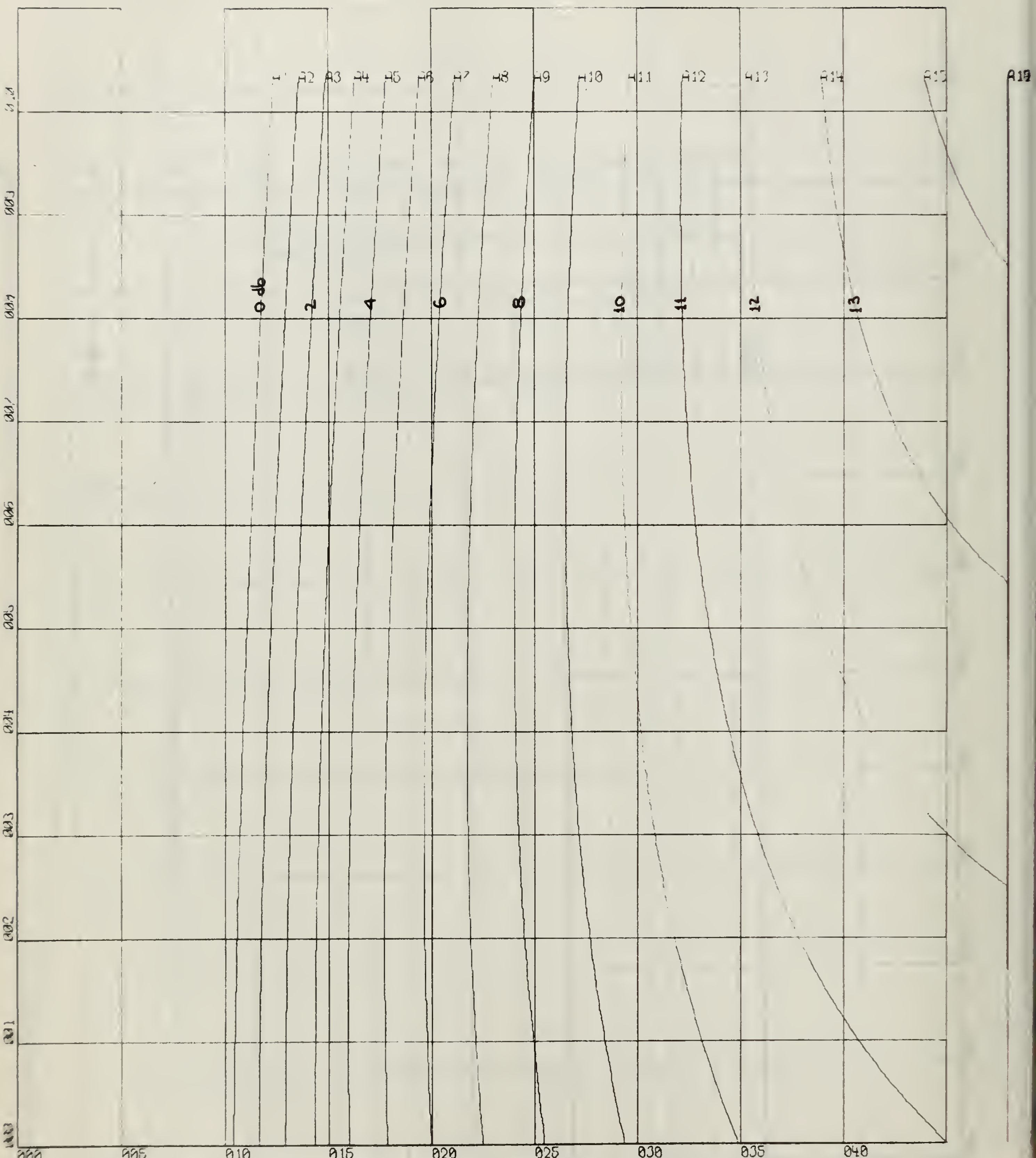

Figure 8-2. Active Filter Constant Bandwidth Curves on Parameter Plane k = 2.0 ω = 0.20

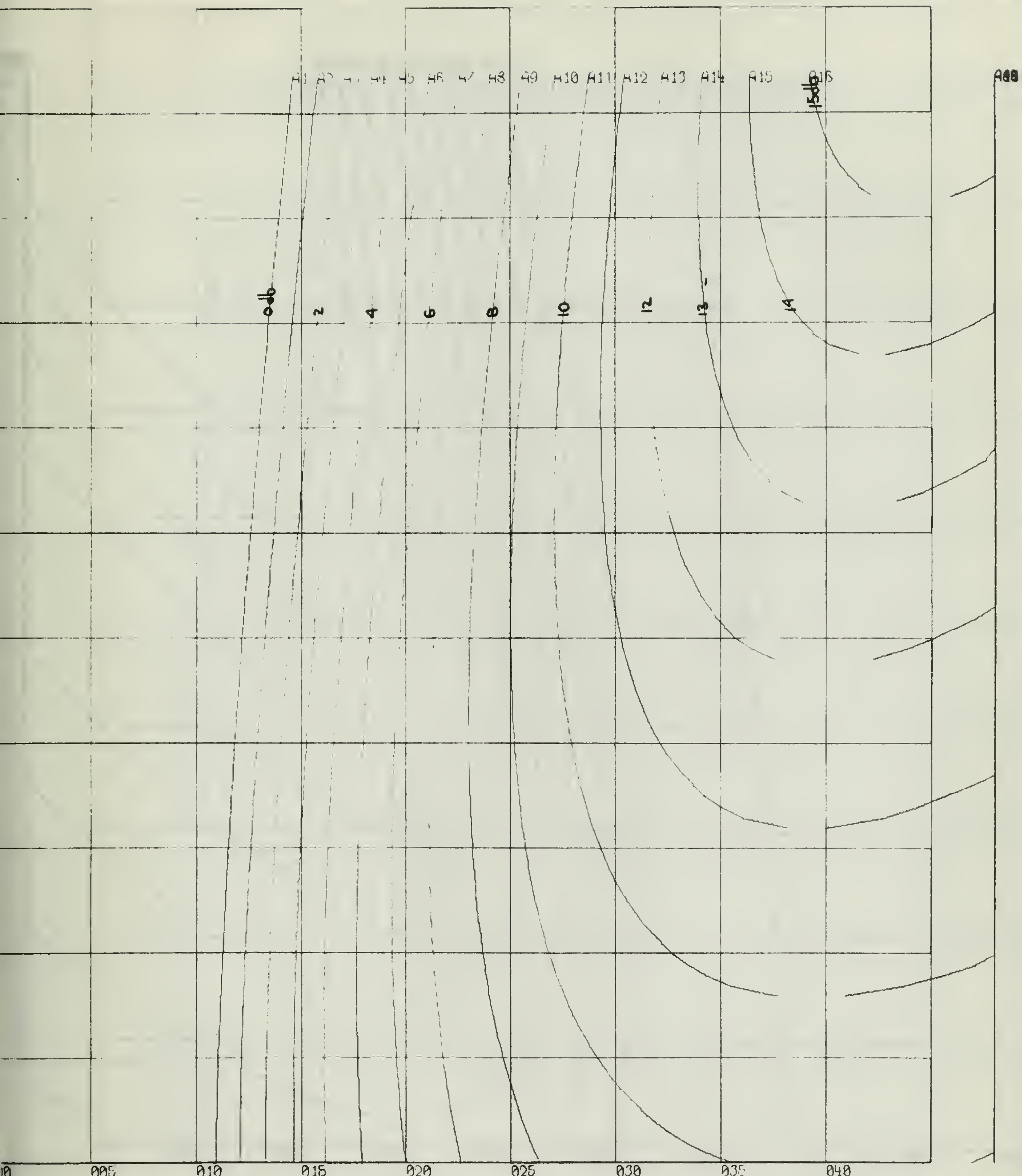

Figure 8-3. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.30

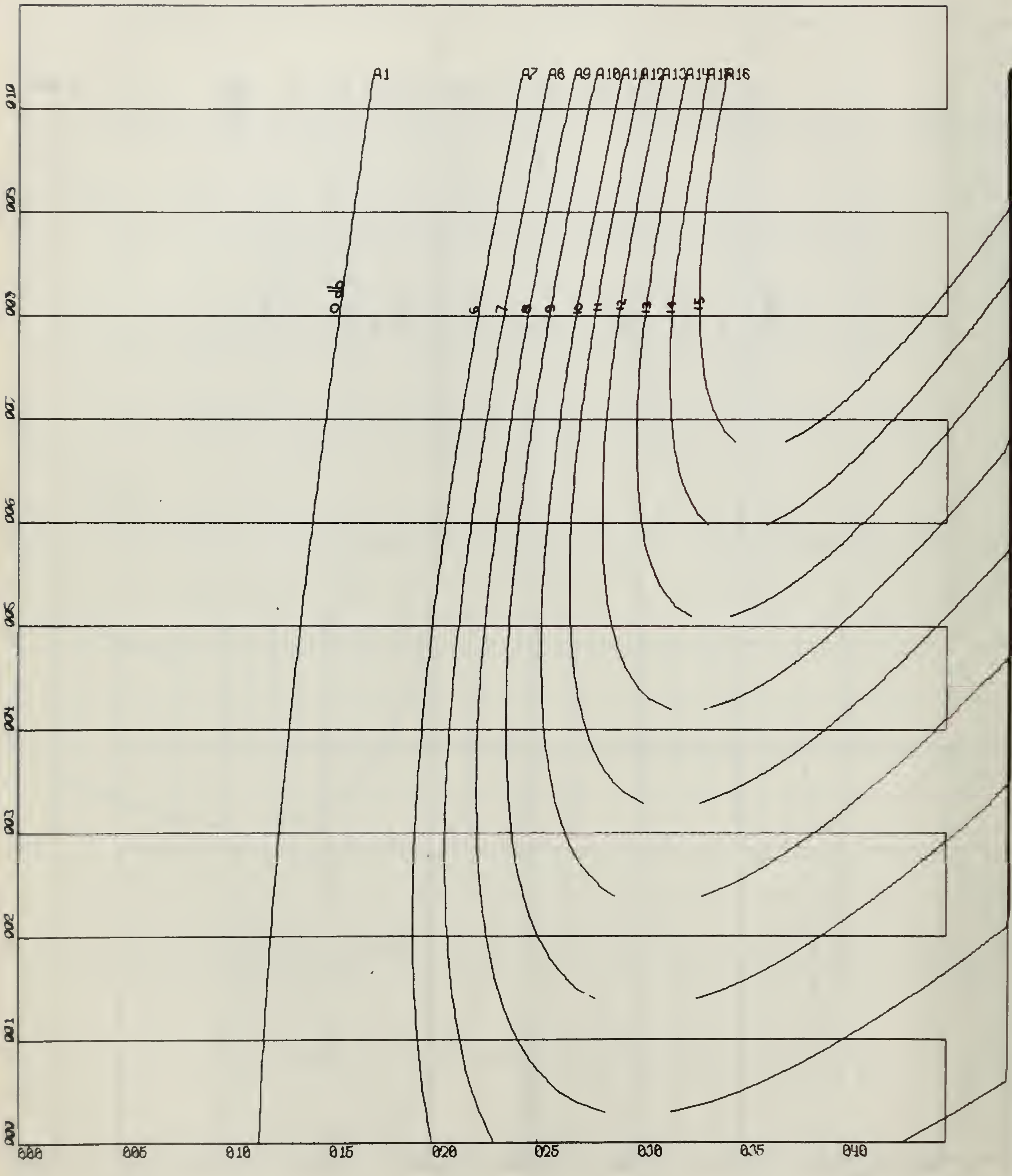

Figure 8-4. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.40

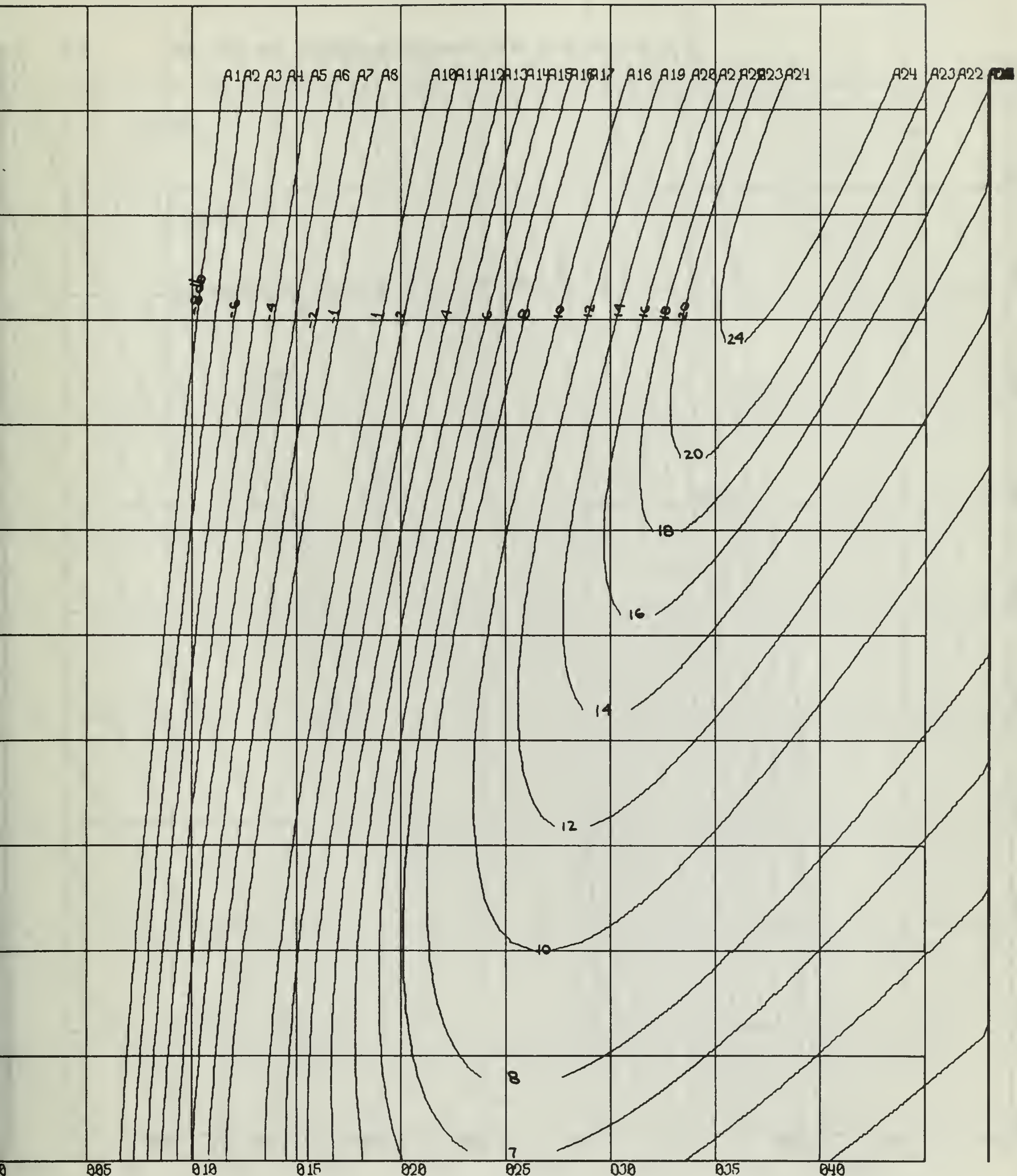

Figure 8-5. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.50

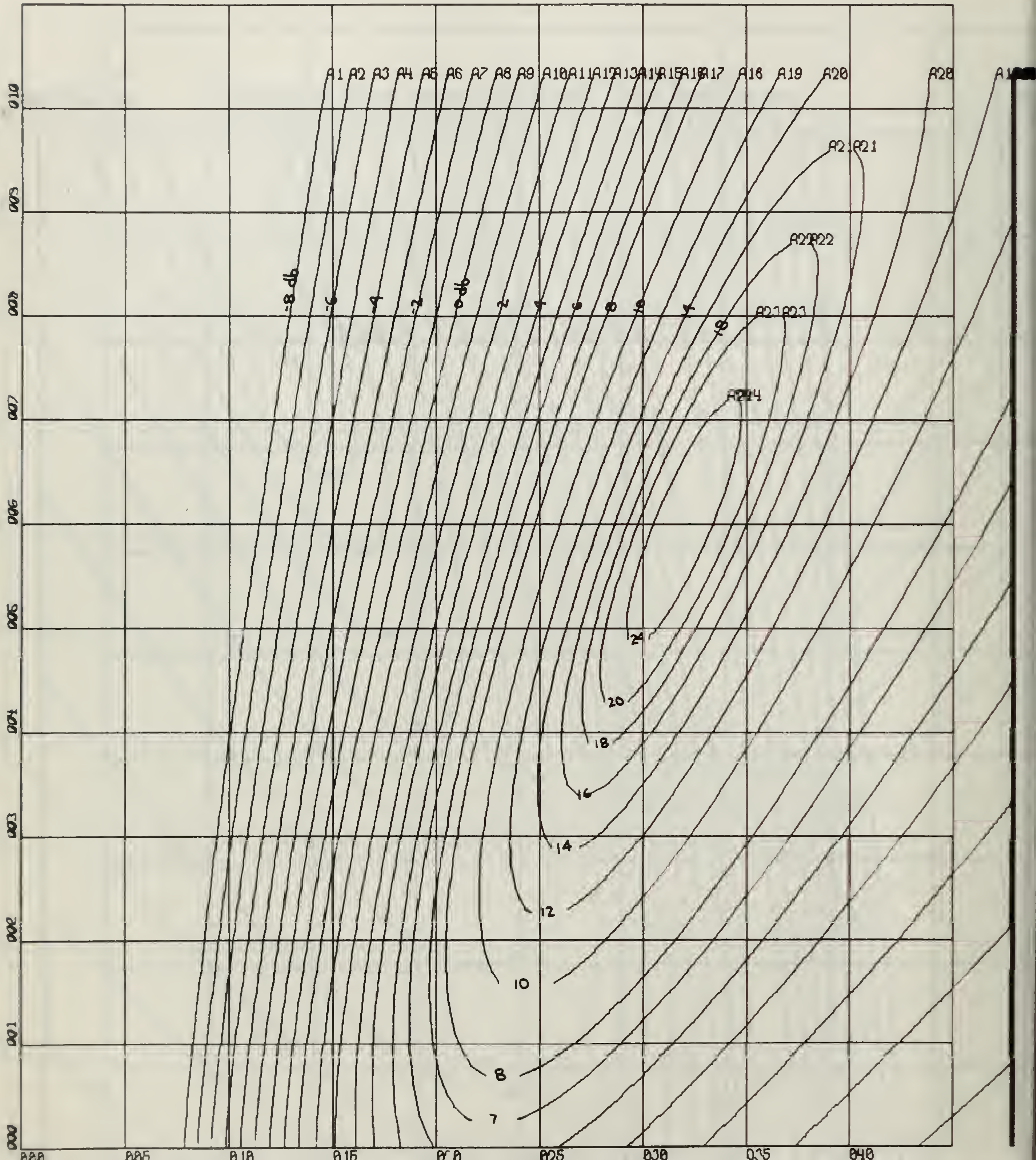

Figure 8-6. Active Filter Constant Bandwidth Curves on Parameter
k = 2.0 ω = 0.60

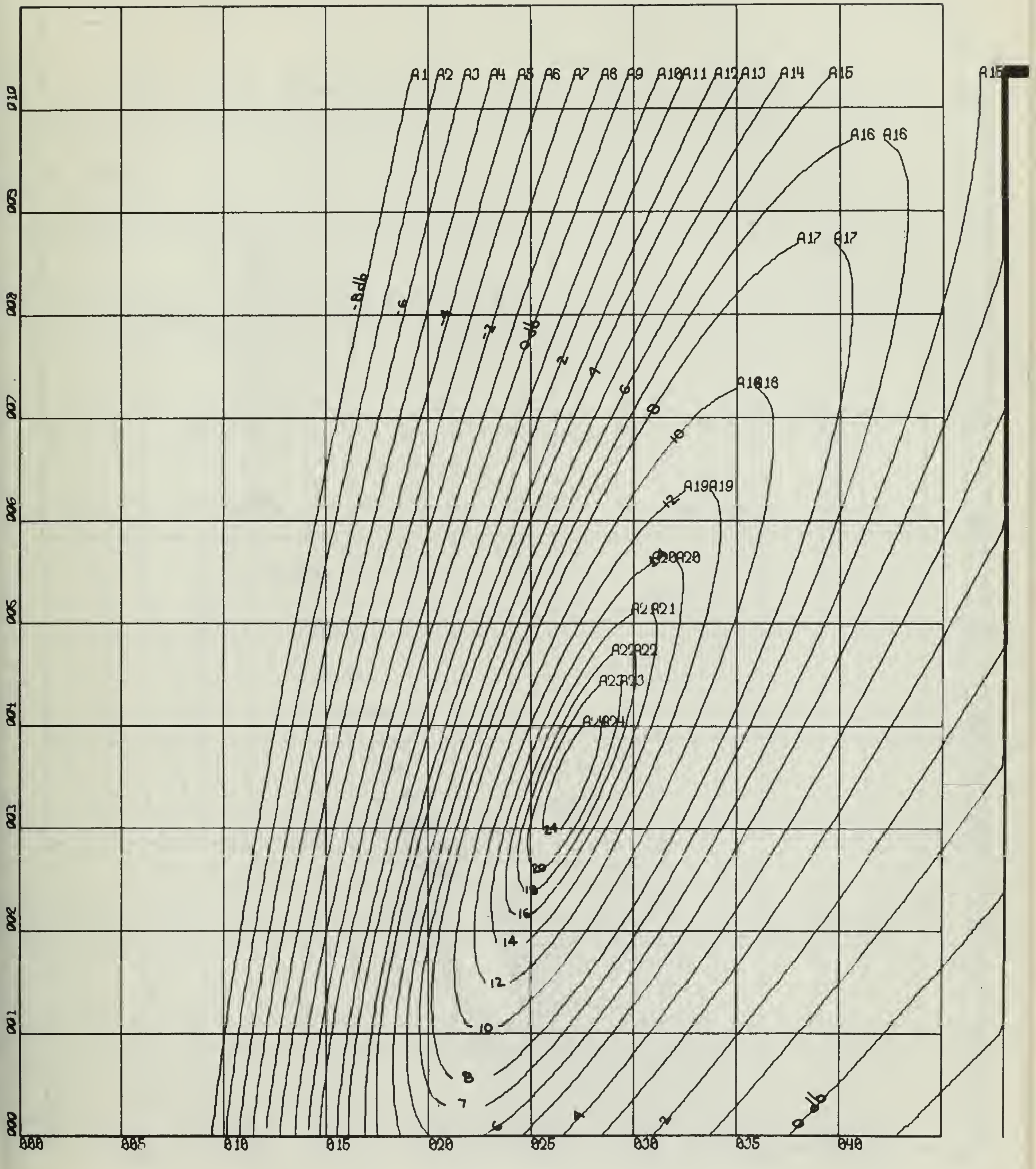

Figure 8-7. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.70

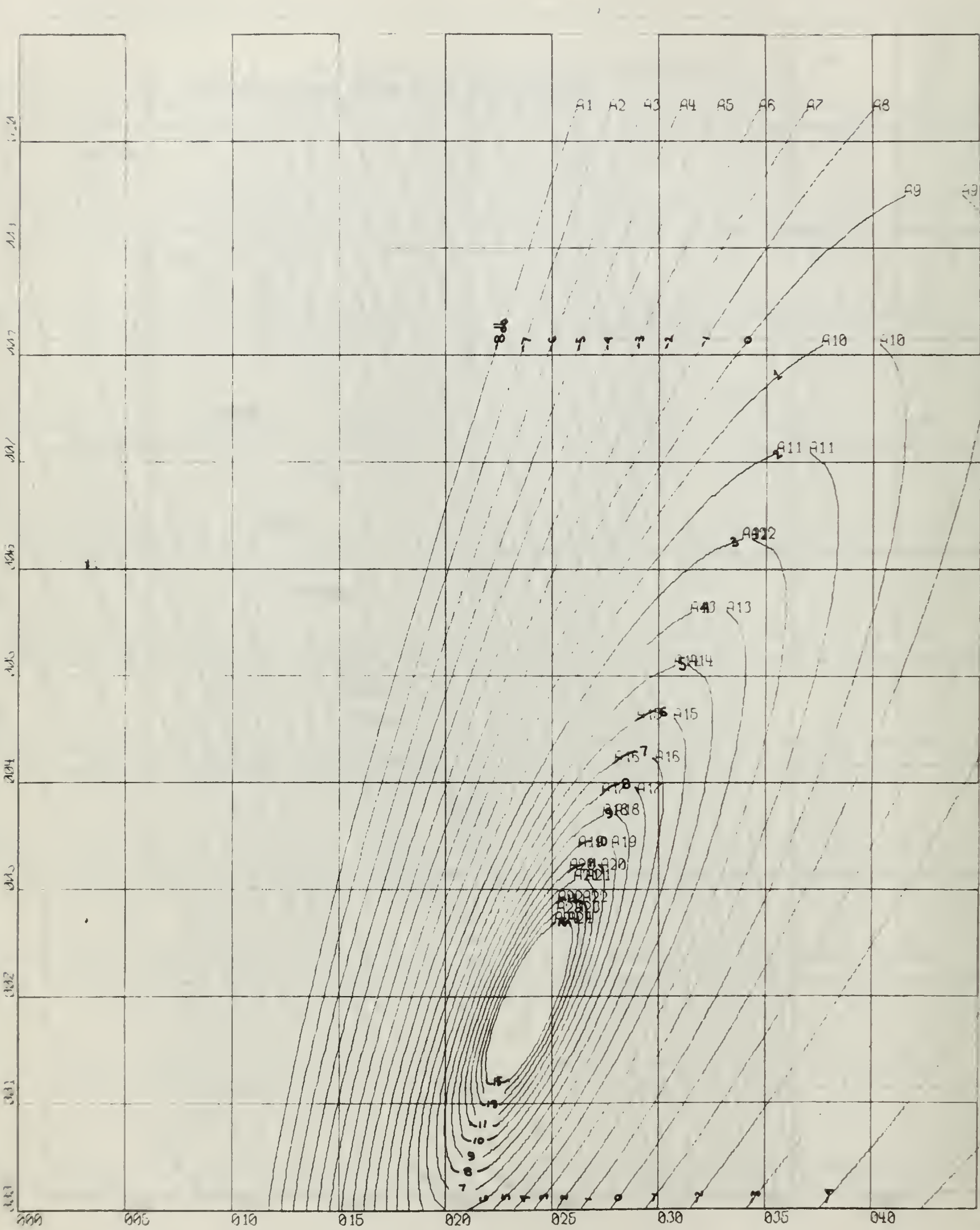

Figure 8-8. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.80

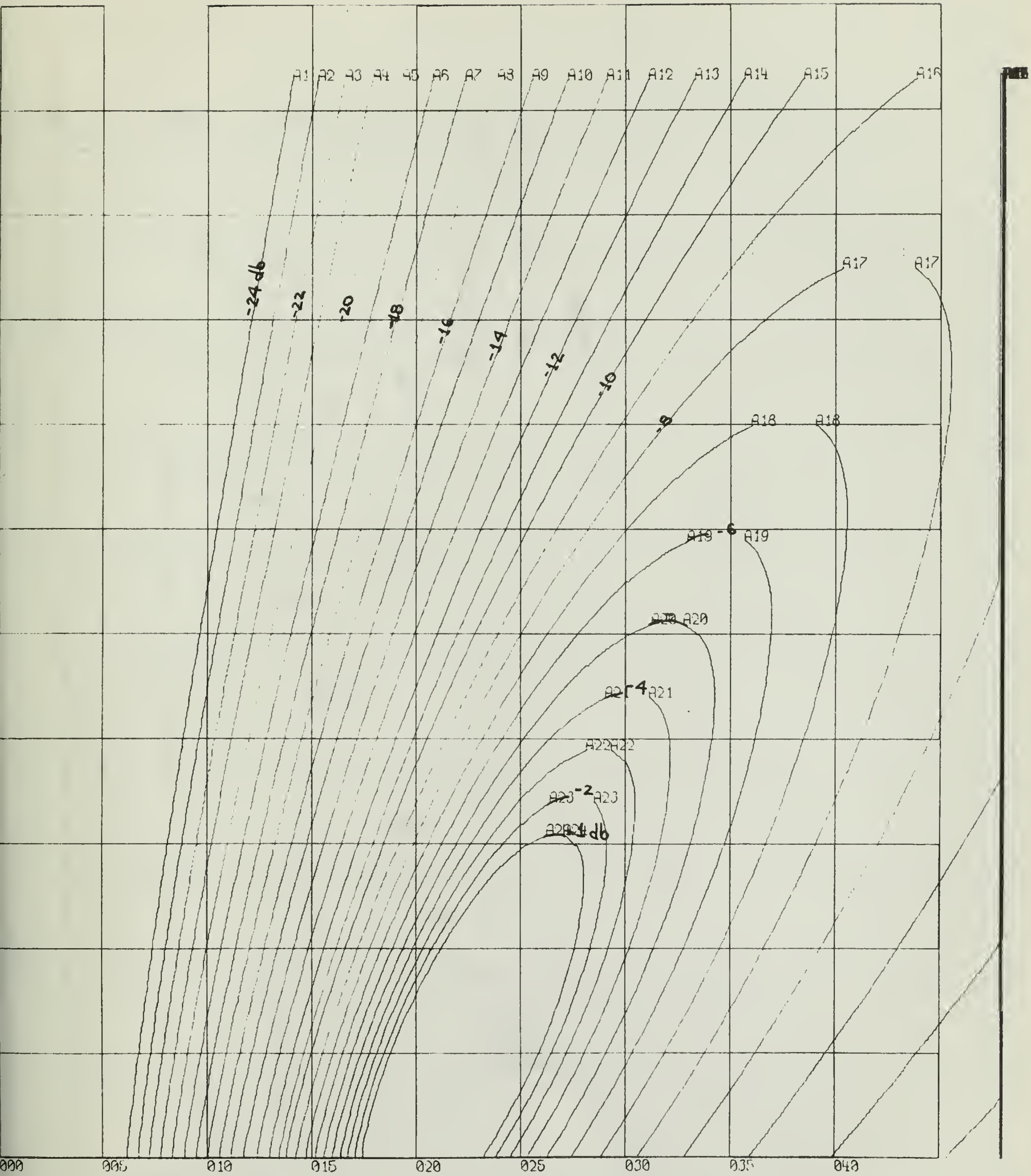

Figure 8-9. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.90

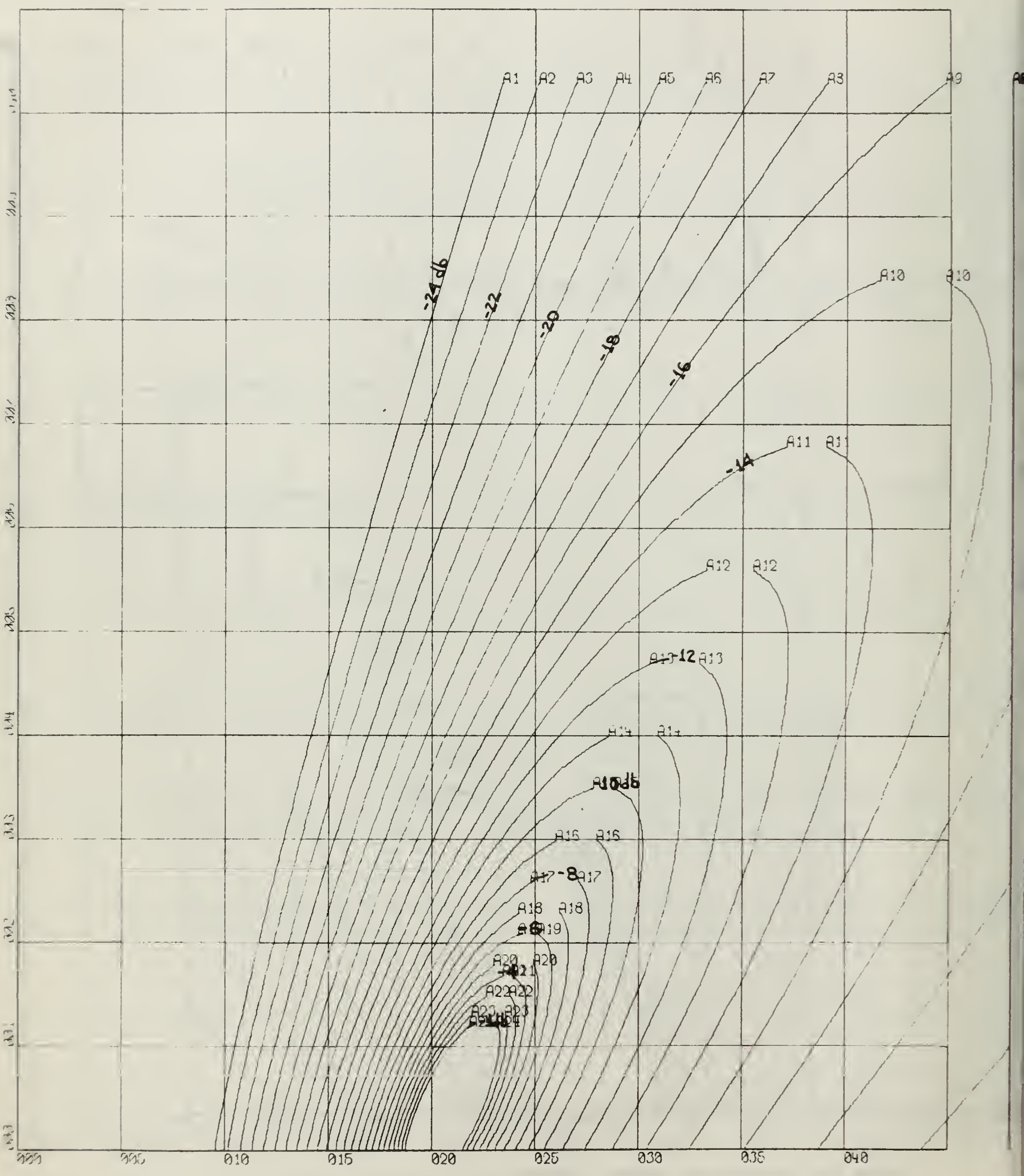

Figure 8-10. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 0.95

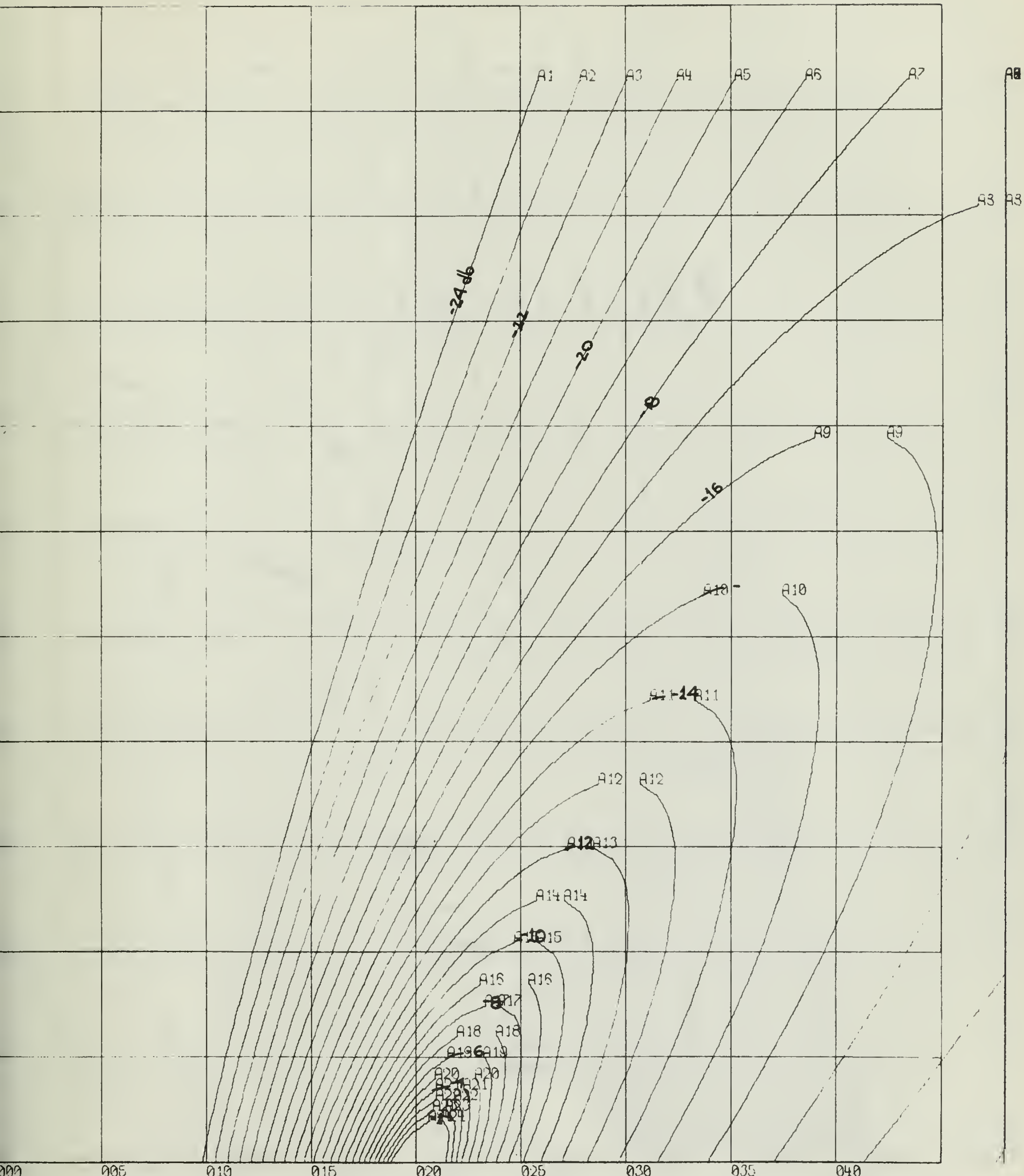

Figure 8-11. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.05

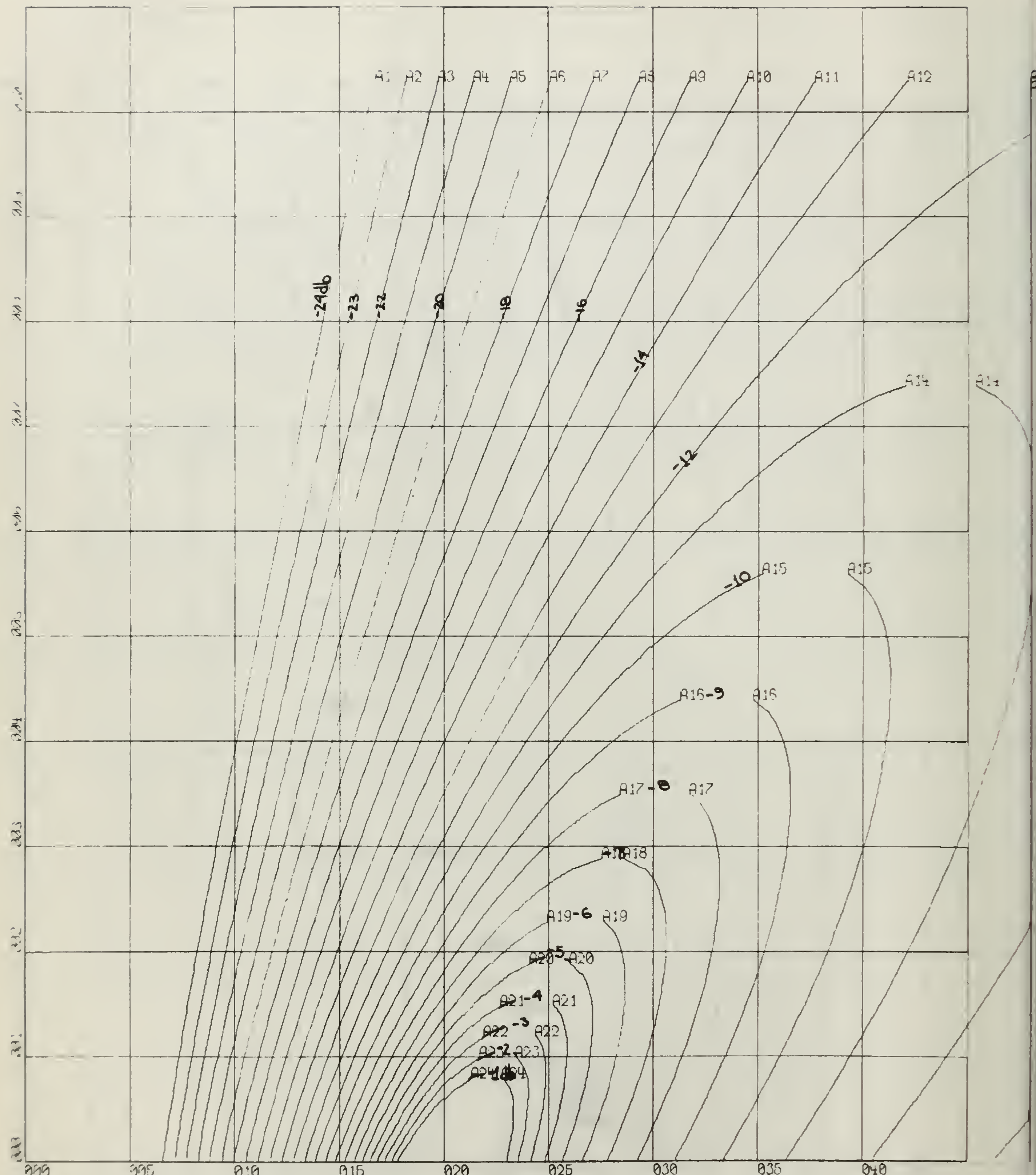

Figure 8-12. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.10

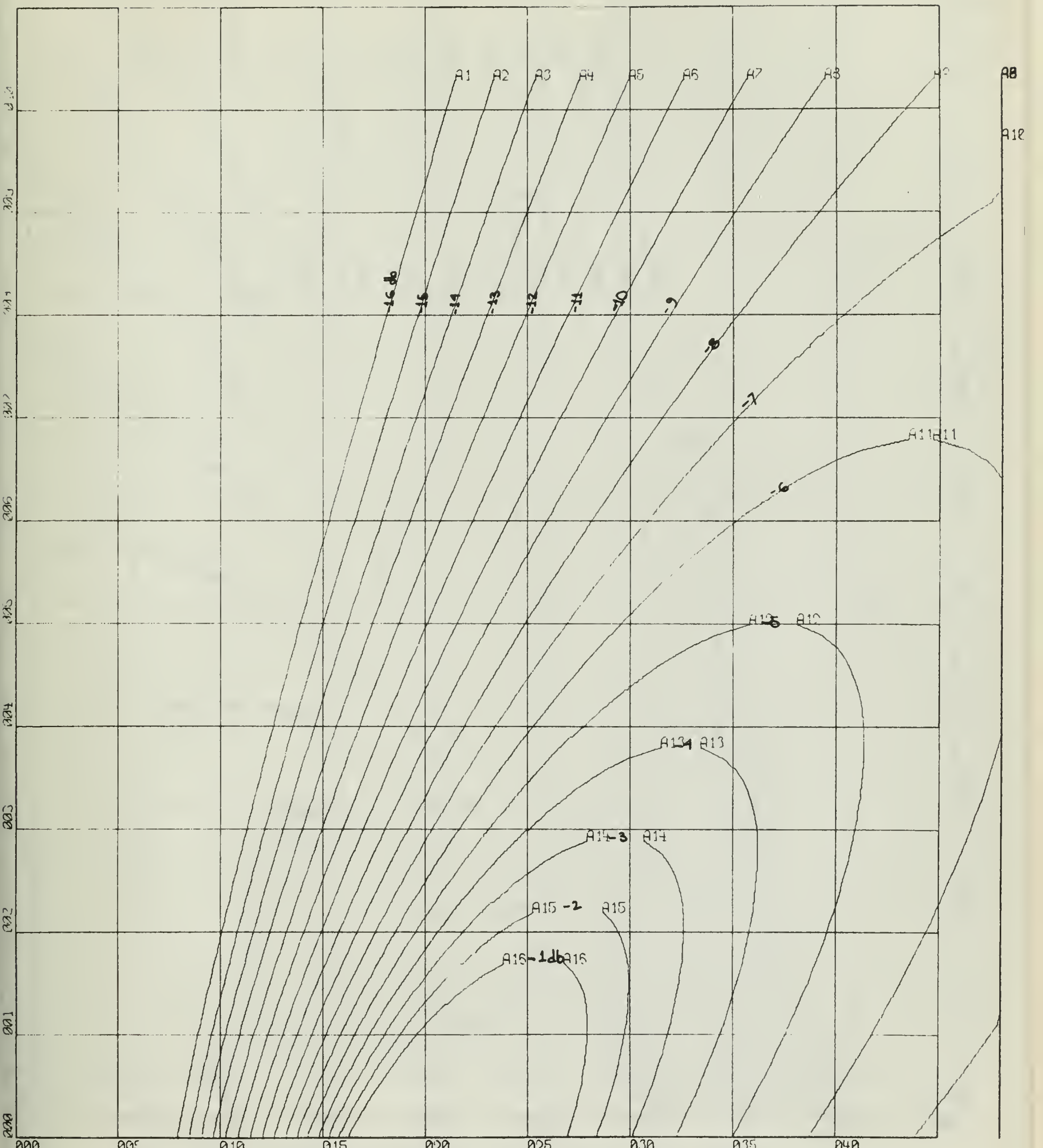

Figure 8-13. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.20

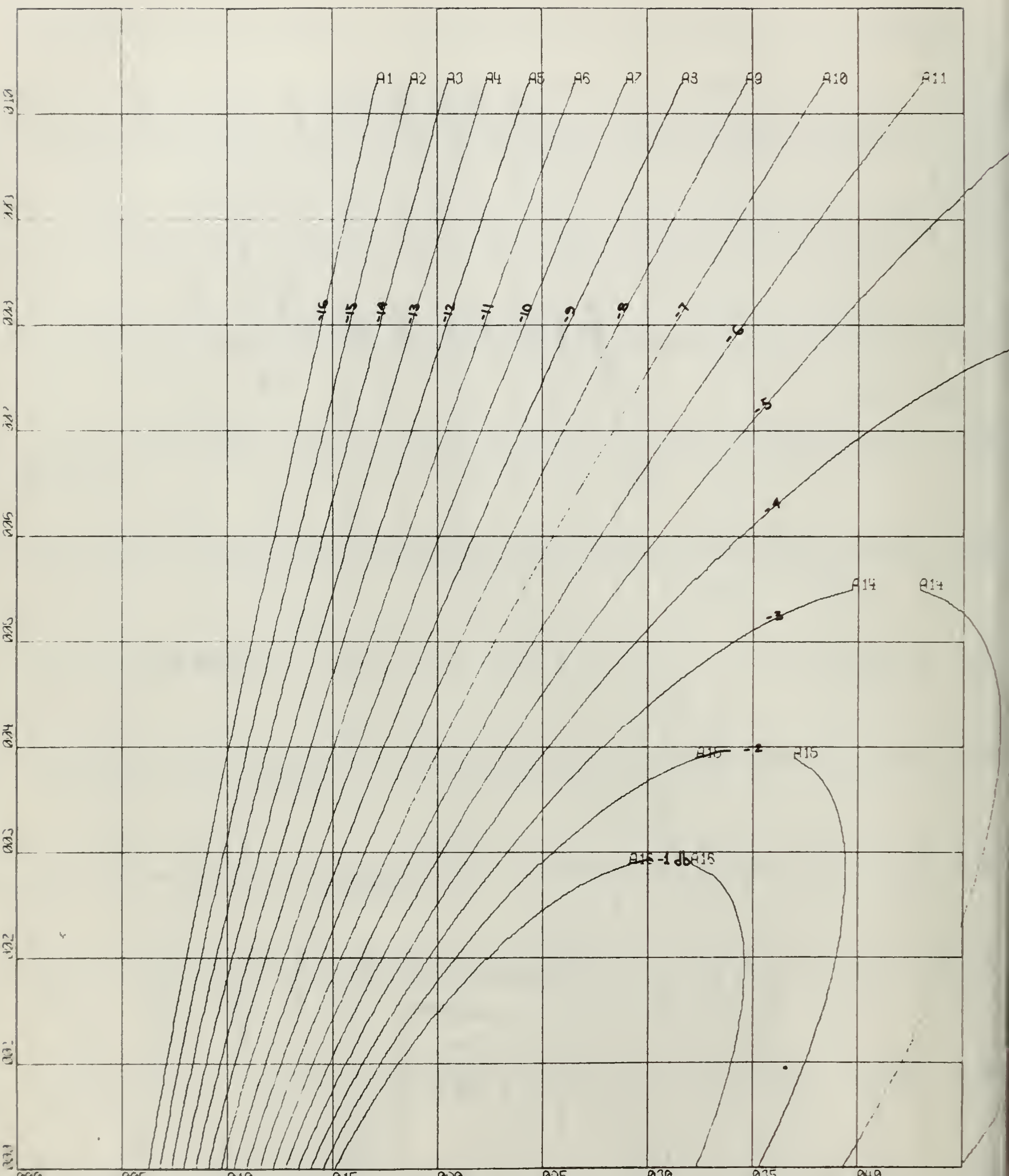

Figure 8-14. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.30

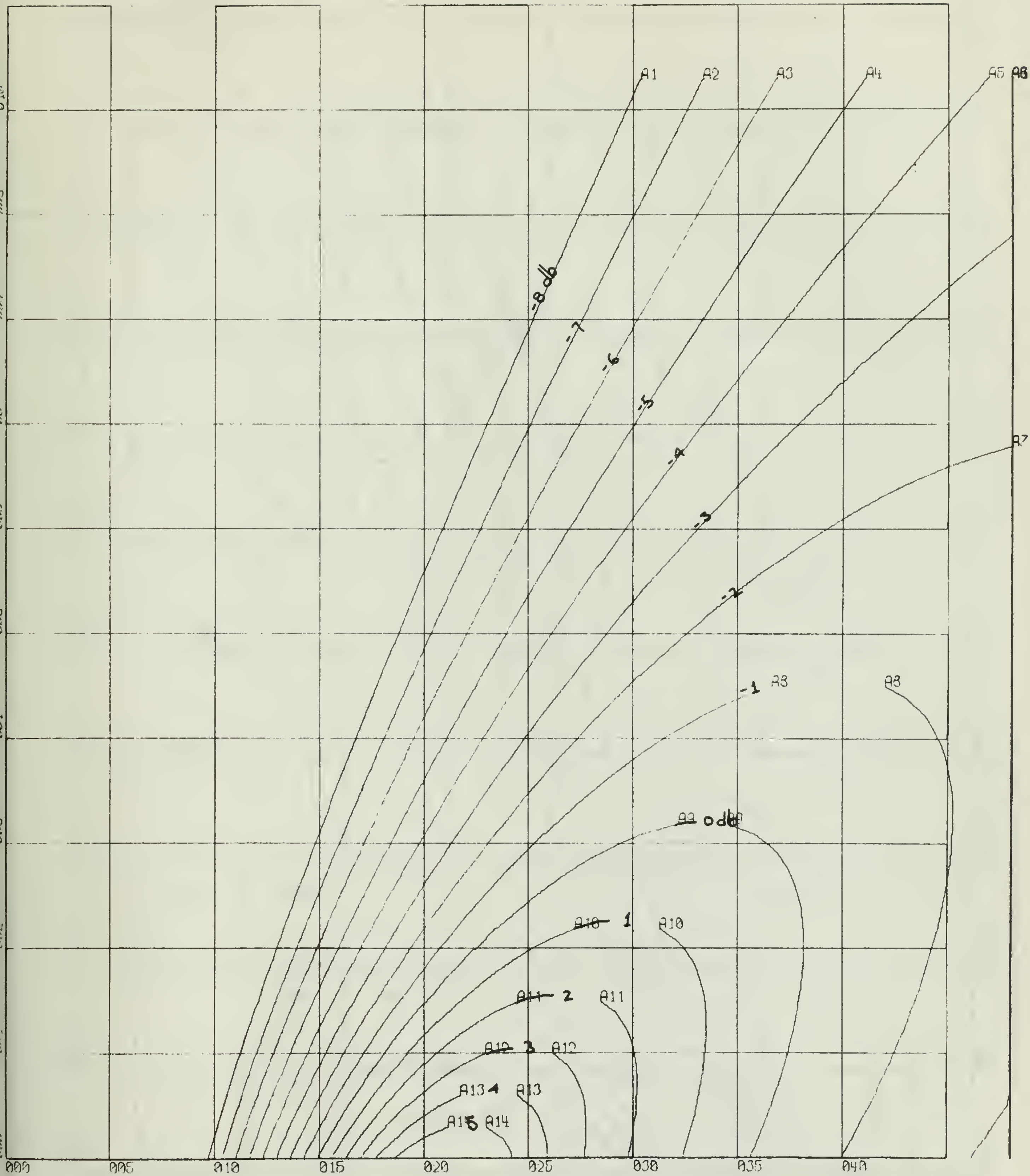

Figure 8-15. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.40

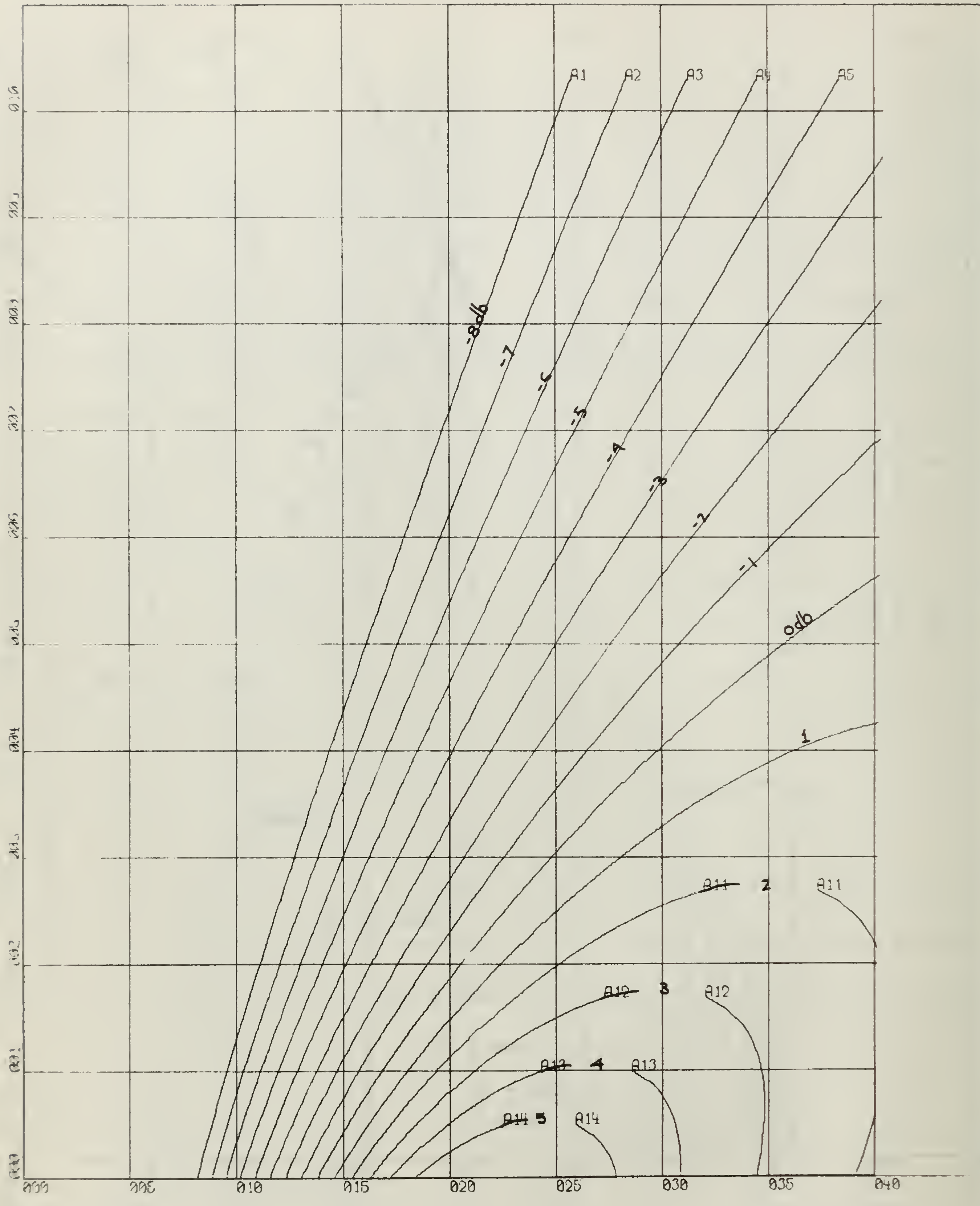

Figure 8-16. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.60

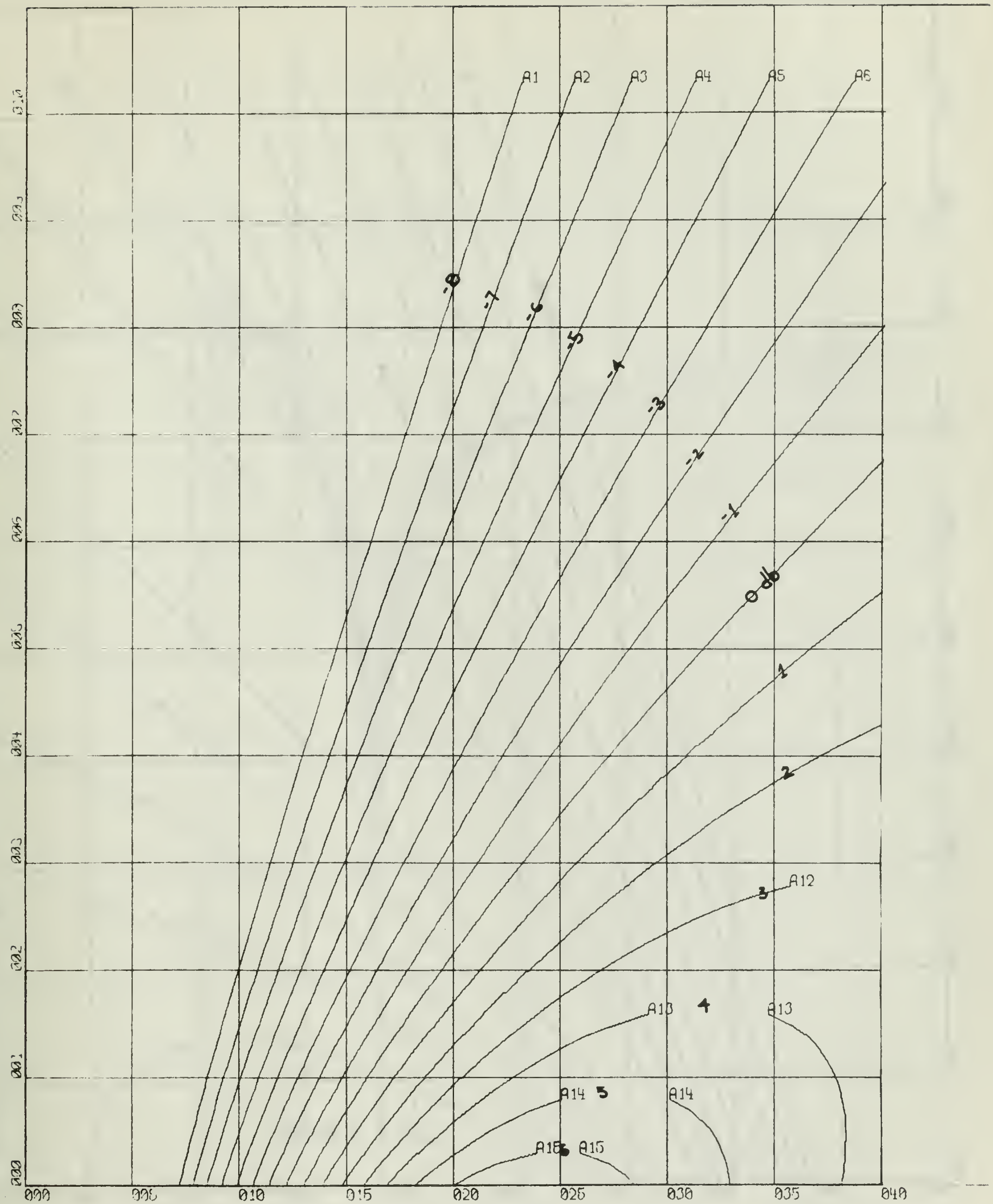

Figure 8-17. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0 ω = 1.80

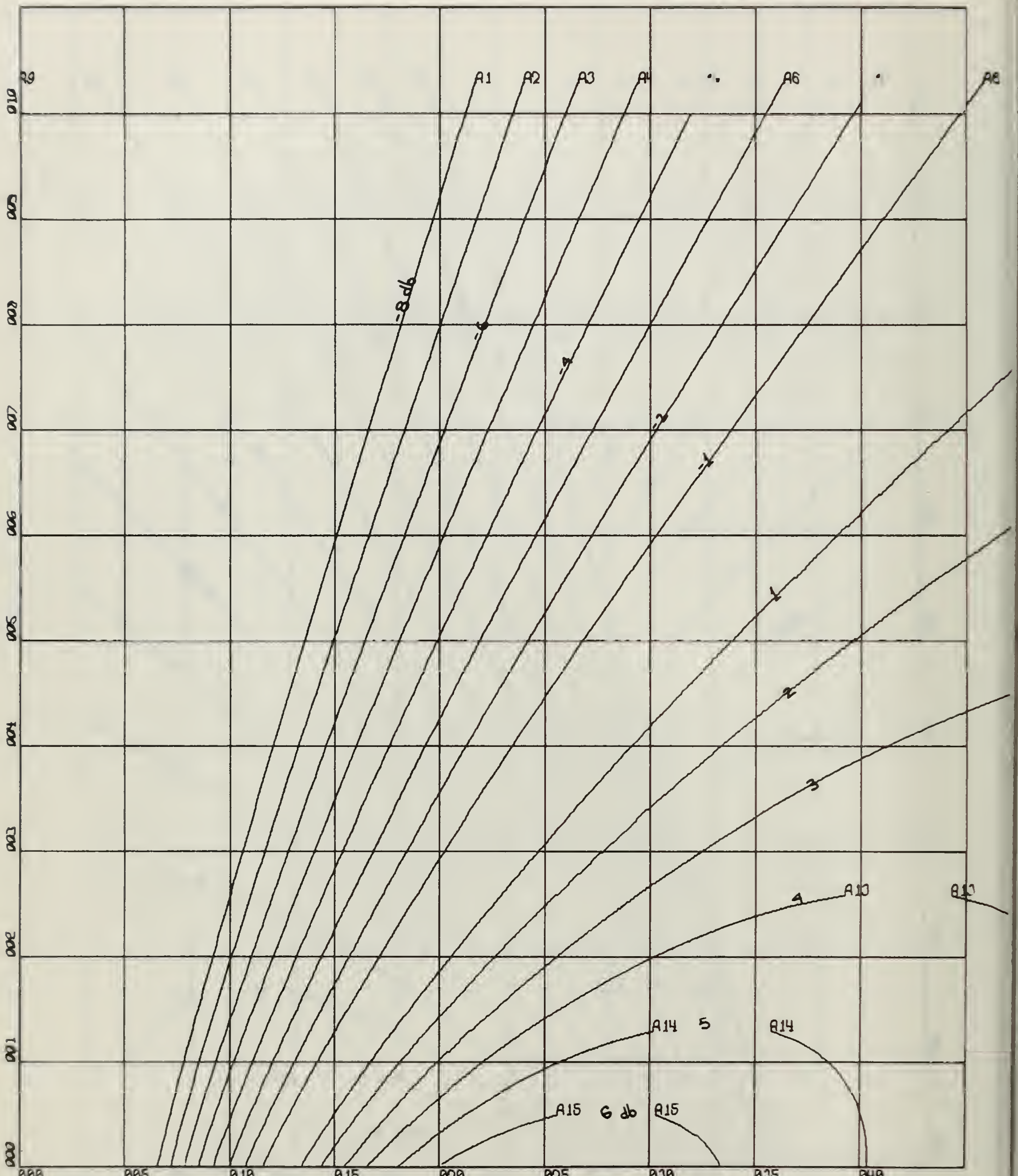

Figure 8-18. Active Filter Constant Bandwidth Curves on Parameter Plane
k = 2.0   C = 0.20

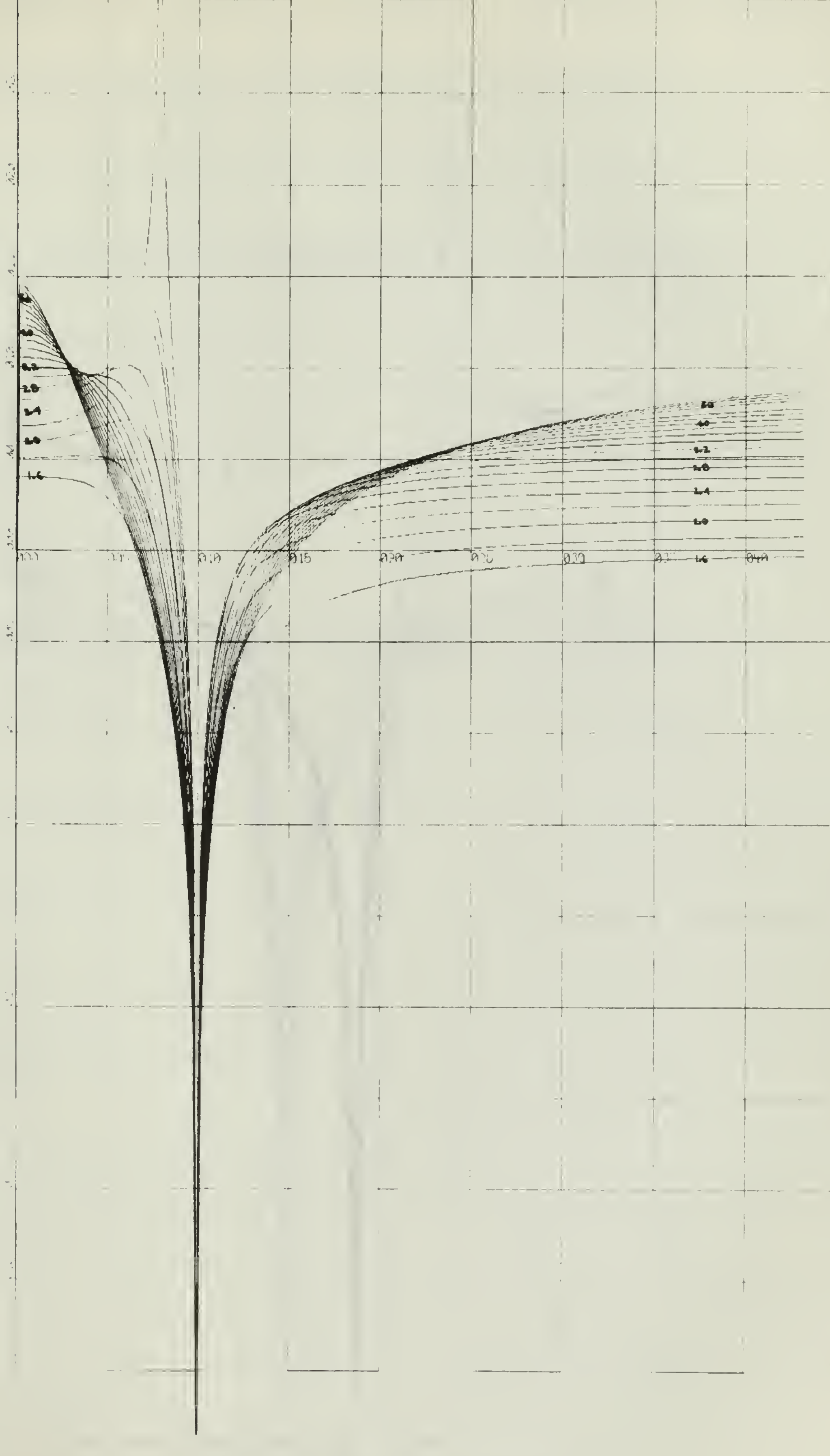

Figure 9-1. Active Filter Frequency Response Curves
k = 2.0 C = 0.20

X Variable Normalized Radian Frequency
X Scale 1" = 0.5
Y Variable Magnitude of Output, M (db)
Y Scale 1" = 5.0 db

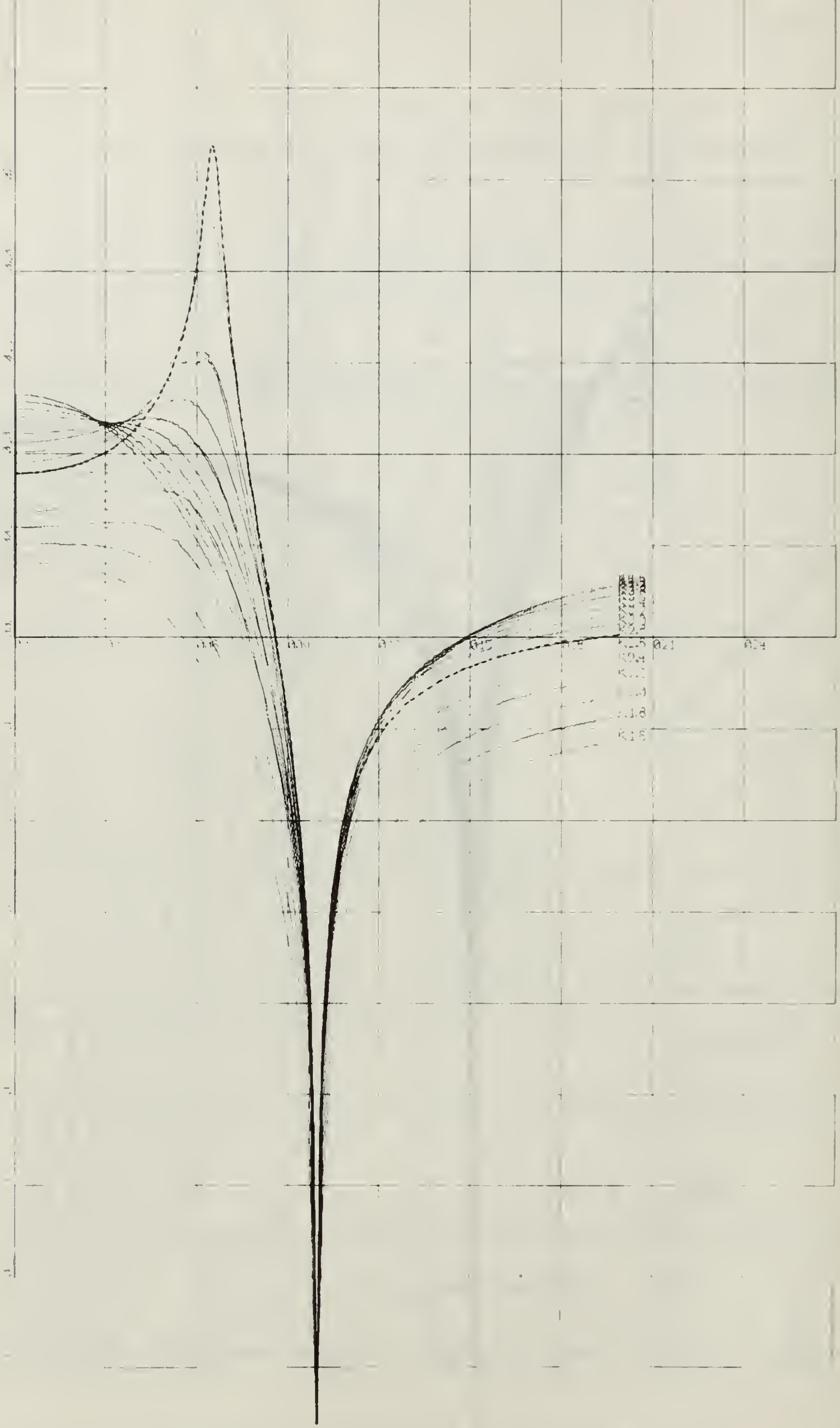

Figure 9-2. Active Filter Frequency Response Curves
k = 2.0 C = 0.436

X Variable Normalized Radian Frequency
X Scale 1" = 0.5
Y Variable Magnitude of Output, M (db)
Y Scale 1" = 5.0 db

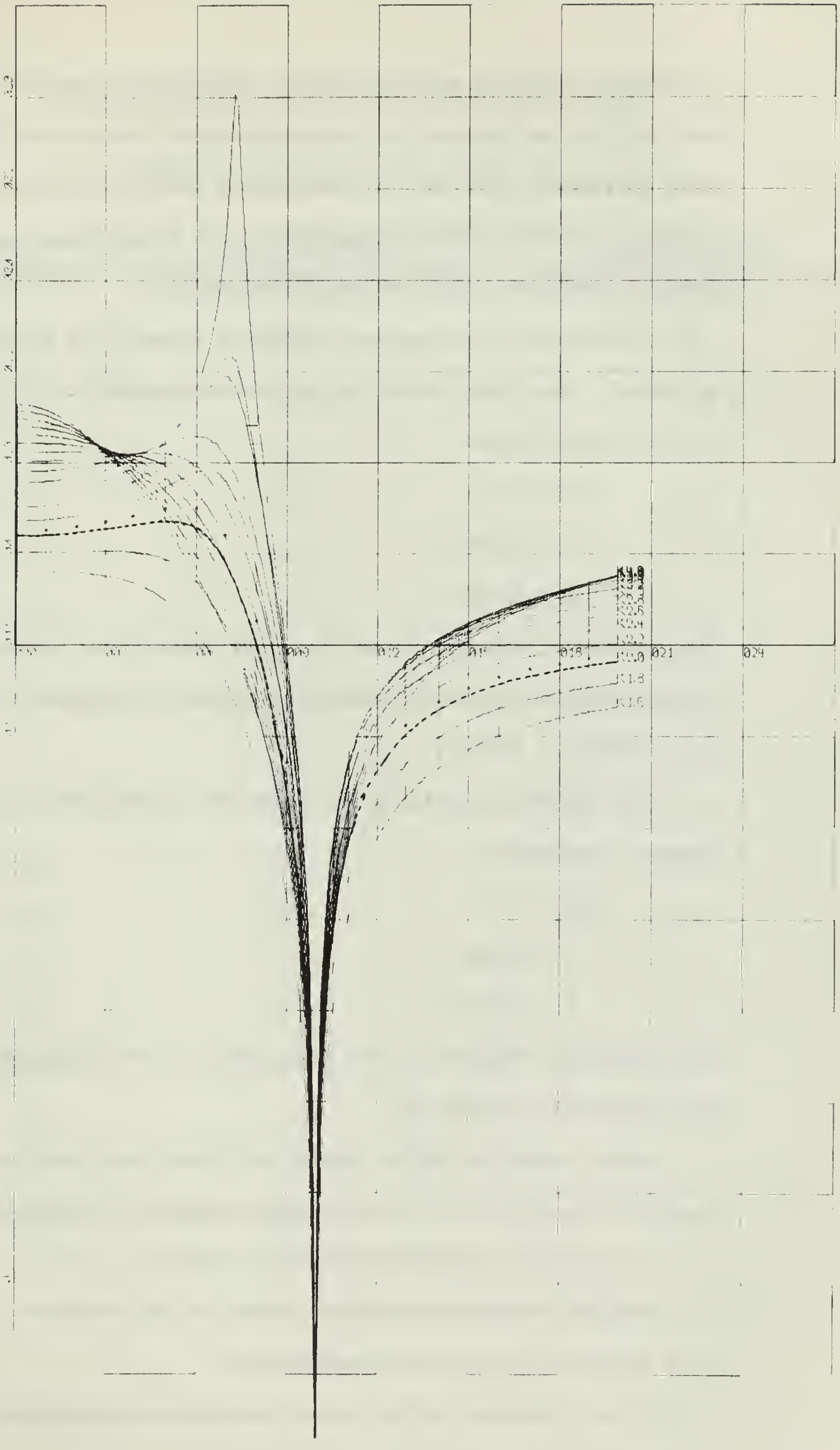

Figure 9-3. Active Filter Frequency Response Curves
k = 2.0 C = 0.284

X Variable Normalized Radian Frequency
X Scale 1" = 0.5
Y Variable Magnitude of Output, M (db)
Y Scale 1" = 5.0 db

a specific value of amplifier gain. At small values of ω and again for $\omega \gg 1.0$ we observe the expected result that output increases as gain increases. But the distinguishing feature of this family of curves is that near the "notch frequency" of 1.0 a resonance peak is observed at some intermediate value of amplifier gain.

To evaluate the constant bandwidth curves, two operating points were selected. The first operating point corresponds to Circuit 2 in Figure 1. For this circuit

$$SK = 2.0$$

$$C = 0.436$$

$$K = 2.762$$

The frequency response curves for this circuit are given in Figure 9-2. A comparison of constant bandwidth against the frequency response values is provided in Table I.

The second operating point selected corresponds to Circuit 3 of Figure 1 for which

$$SK = 2.0$$

$$C = 0.284$$

$$K = 2.074$$

The frequency response curves for Circuit 3 are displayed in Figure 9-3 and tabulated in Table II.

Other operating points could well have been selected, but further repetition would do little more than reemphasize the conclusions that are arrived at by considering Tables I and II. For it is clearly apparent that the constant bandwidth curves on the parameter plane are excellent predictors of circuit performance.

It is therefore valid to use the constant bandwidth curves for the design of such active filters.

TABLE I - Circuit 2 ($C = 0.436$, $K = 2.762$)

RADIAN FREQUENCY ω	AMPLITUDE FROM CONSTANT BANDWIDTH CURVES A_2	AMPLITUDE FROM FREQUENCY RESPONSE CURVES A_2
0.1	9.0 db	9.0 db
0.2	9.3	9.4
0.3	10.0	10.0
0.4	11.2	11.3
0.5	13.5	13.6
0.6	19.5	20.0
0.7	18.5	19.0
0.8	6.0	6.0
0.9	-4.4	-4.4
0.95	-12.1	-12.5
1.05	-14.7	-14.0
1.10	-9.8	-9.0
1.20	-5.7	-5.6
1.30	-3.7	-3.6
1.40	-2.5	-2.5
1.60	-1.2	-1.1
1.80	-0.4	-0.3

EVALUATION OF "CONSTANT BANDWIDTH" CURVES

TABLE II - Circuit 3 (C = 0.284, K = 2.074)

RADIAN FREQUENCY	AMPLITUDE FROM CONSTANT BANDWIDTH CURVES	AMPLITUDE FROM FREQUENCY RESPONSE CURVES
ω	A_3	A_3
0.1	6.2 db	6.2 db
0.2	6.5	6.5
0.3	6.8	6.8
0.4	7.1	7.1
0.5	7.5	7.5
0.6	7.8	7.6
0.7	6.0	6.0
0.8	1.8	1.4
0.9	-6.1	-6.4
0.95	-13.7	-14.0
1.05	-15.7	-15.0
1.11	-10.6	-10.6
1.20	-6.3	-6.5
1.30	-4.3	-4.3
1.40	-3.0	-3.1
1.60	-1.8	-1.8
1.80	+0.1	-1.0

5. Active Filter Constant Zeta and Omega Curves on the Parameter Plane.

The computer programs developed by Nutting were the basis for the constant zeta and omega curves displayed on the parameter plane of Figure 10. The transfer functions used for this parameter plane plot are derived from the active filter sections that have been designated circuit #2 and circuit #3.

By careful selection of the proper values of amplifier gain, K, and the normalized input capacitance, C, it should be possible to adjust the root locations to any desired value of damping ratio, zeta, and radian frequency, omega, displayed on the parameter plane of Figure 10.

5.1 Pole Locations and the Design of the Low Pass Filter

The ideal low pass filter which is frequency response optimized has a gain vs. frequency characteristic shown in Figure 11.

Although such filter performance is not physically realizeable, there are many filters that provide acceptable approximations to the ideal. Two mathematically designed filters, the Butterworth and the Chebyshev, will be discussed here.

The Butterworth filter is said to be optimally flat at zero frequency, with amplitude related to frequency by

$$|A|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}$$

where

$|A|$ is the amplitude

ω_c is the cut-off frequency

n is the order of the filter transfer function

This relationship is plotted in Figure 12.

The frequency response of the Chebyshev filter shown in Figure 13

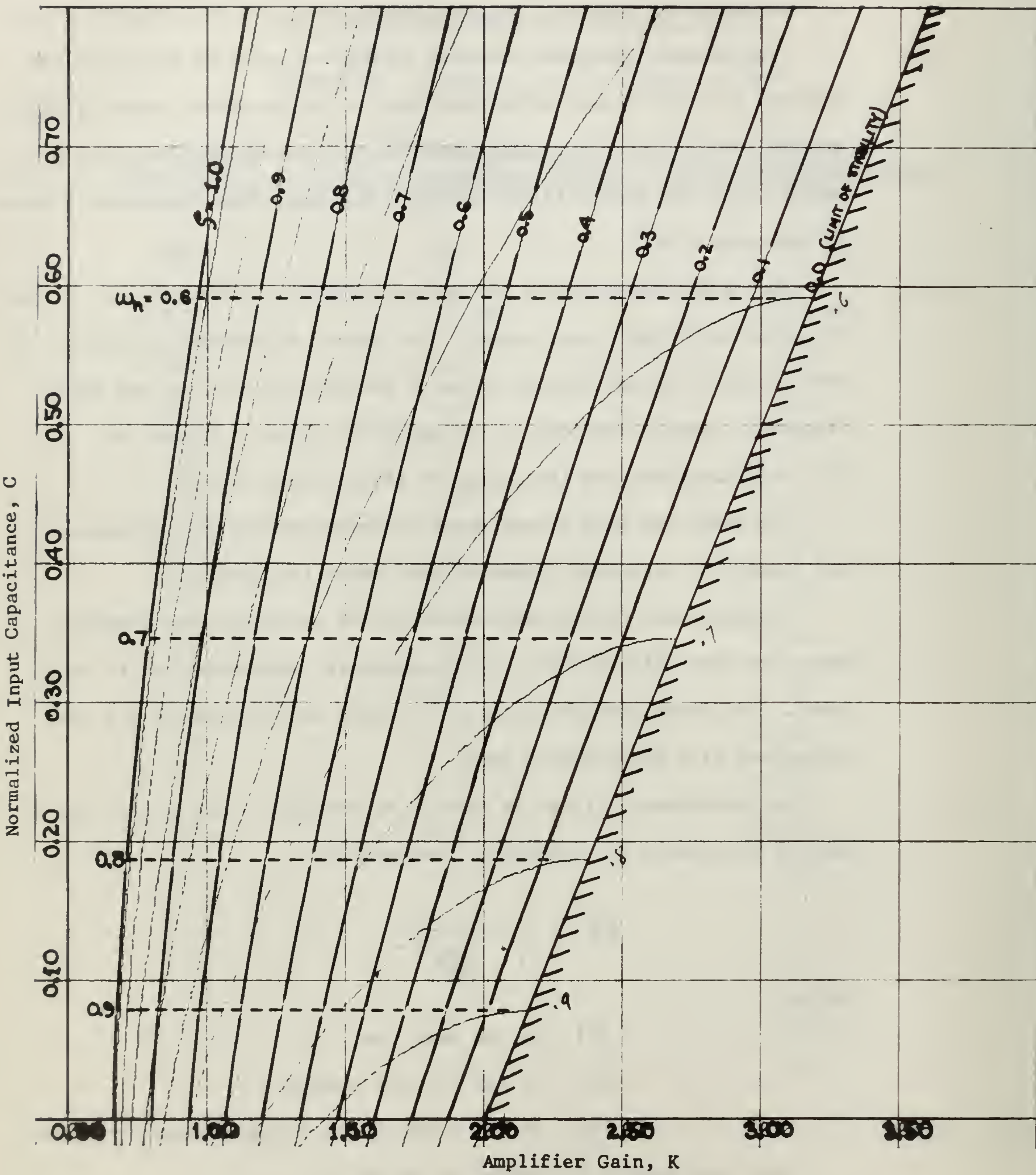

Figure 10. Constant Zeta and Omega Curves on the Parameter Plane

approximates the ideal frequency response with a "constant ripple" response in the pass band, confining the error to some specified value throughout the pass band. It should be noted that the Chebyshev filter places equal emphasis on all frequencies in the pass band, whereas the Butterworth filter emphasizes the near zero frequency behavior.

To achieve the frequency response shown in Figure 12, the poles of the Butterworth filter transfer function displayed on the s-plane are uniformly spaced on a semicircle of radius ω_c as shown in Figure 14.

In a similar fashion the frequency response of the Chebyshev filter is accomplished by placing the poles of the filter transfer function on an ellipse on the s-plane. It has been shown that the proper positioning of the poles can be related to the poles of a Butterworth filter of the same order by reducing the real component of the respective roots by the factor tanh β [6], without modifying the imaginary component. Thus

$$\sigma_{\text{Chebyshev}} = (\sigma_{\text{Butterworth}})(\tanh \beta)$$

where

$$\beta \triangleq \frac{1}{n} \sinh^{-1} \frac{1}{\epsilon}$$

n is the order of the filter

ϵ is related to the amplitude of the ripple with $\frac{1}{\sqrt{1 + \epsilon^2}}$ the minimum amplitude in the pass band as shown in Figure 13.

5.2. Manipulation of Circuit Poles into Chebyshev Configuration

The poles of the active filter system shown in Figure 1 will now be modified utilizing parameter plane techniques to form a Chebyshev configuration of poles.

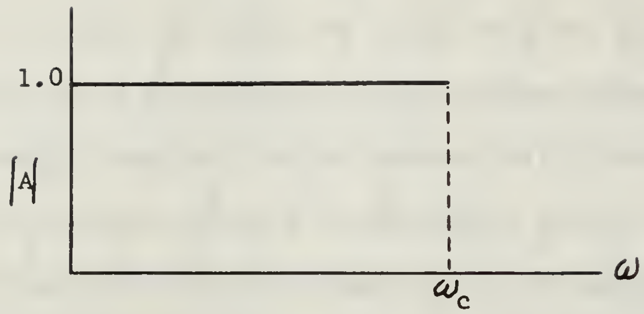

Figure 11. Ideal Low Pass Filter Frequency Response

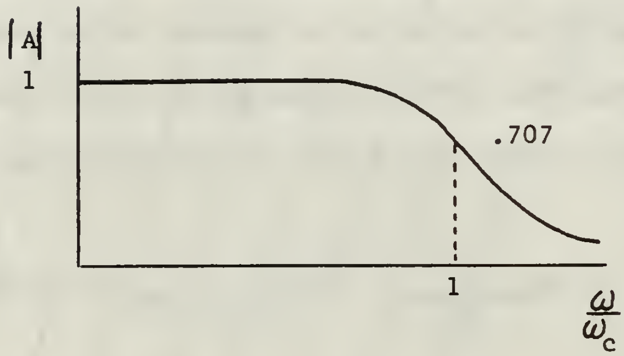

Figure 12. Butterworth Filter Frequency Response

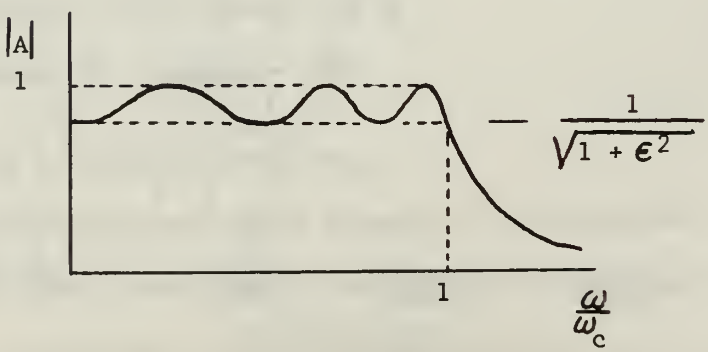

Figure 13. Chebyshev Filter Frequency Response

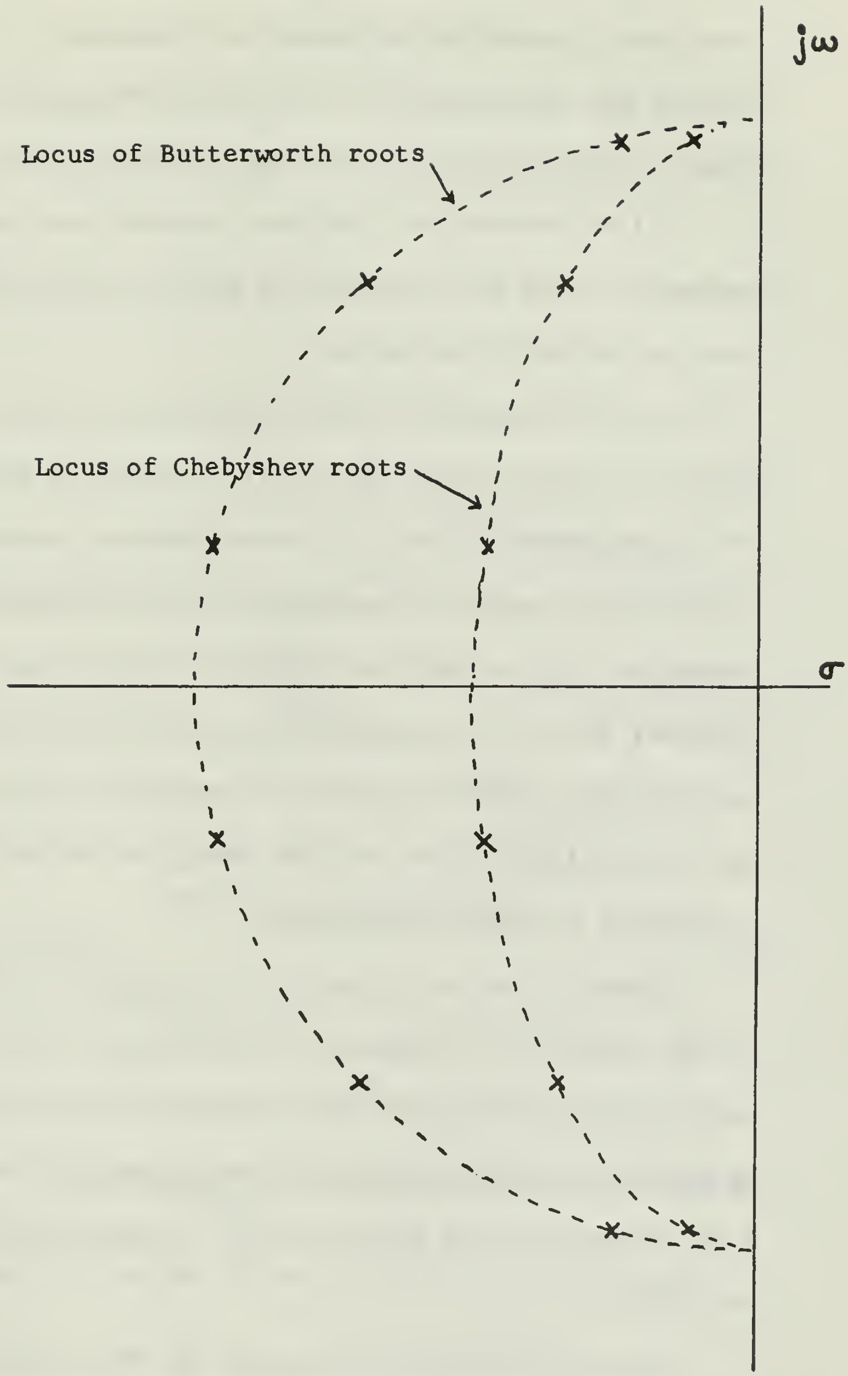

Figure 14. Relative Position of Poles for Butterworth and Chebyshev Filters (6^{th} Order)

The purpose of this exercise is two-fold:

(1) To illustrate the methods required for proper pole relocation which must include an adjustment of frequency scaling factors introduced through the normalization of the notch frequencies that was one of the steps in producing the normalized parameter plane of Figure 10.

(2) To demonstrate that the constant zeta and omega curves on the parameter plane are an effective means of obtaining a desired performance from an active filter system.

For the purposes of this discussion the roots associated with the low pass filter circuit #1 will be assumed to be fixed and the positions of the remaining poles will be manipulated to obtain the Chebyshev configuration. Another assumption is that the notch or "zero output" frequency is high so that the effects of the transfer function zero will be ignored, that is, only the poles will be considered. The two assumptions are made for the convenience of clarifying the following presentation. No serious limitations upon the design of active filter systems are introduced by these assumptions.

Figure 15 is an s-plane plot of the pole configuration of the active filter system. The numbers in parentheses identify the circuit number which is associated with that particular pole pair. Thus the poles nearest the negative real axis designated (1) are associated with circuit #1 and form the basis for the placement of the remaining two sets of poles.

Since the imaginary component of this pole pair is equal to the imaginary component of the corresponding Butterworth filter, the poles designated (1) establish the radius, ω_0, of the reference Butterworth semicircle.

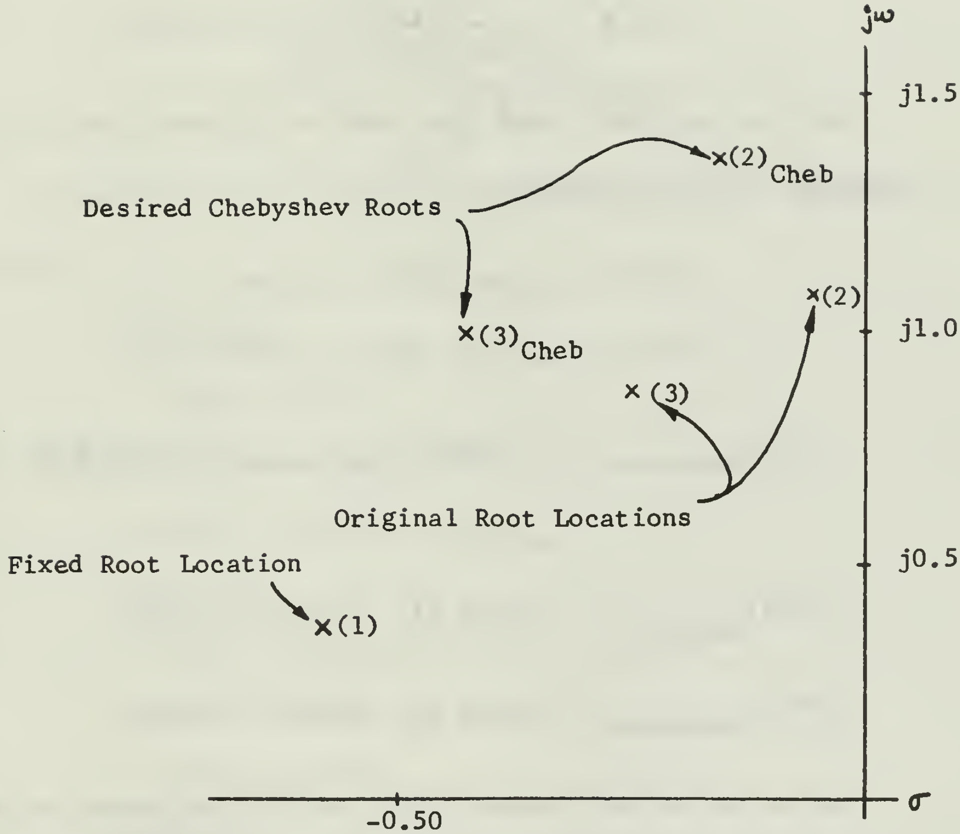

Figure 15. Active Filter System Pole Configuration

$$\omega(1) = \omega_o \sin 165^\circ = 0.3638$$

$$\omega_o = \frac{\omega(1)}{\sin 165^\circ} = 1.406$$

Similarly the real component of the poles designated (1) establish the magnitude of the ripple by specifying the value of tanh

$$\tanh\beta = \frac{\sigma_{Chebyshev}}{\sigma_{Butterworth}} = \frac{\sigma(1)}{\omega_o \cos 165^\circ}$$

$$\tanh\beta = \frac{-0.5776}{(-0.9659)(1.406)} = 0.4254$$

Now that ω_o and $\tanh\beta$ have been established, the remaining pole positions can be determined.

$$\omega(3) = \omega_o \sin 135^\circ = 0.994$$

$$\omega(2) = \omega_o \sin 105^\circ = 1.358$$

$$\sigma(1)_{Chebyshev} = \tanh\beta\,\sigma(1)_{Butterworth} = \tanh\beta\,\omega_o \cos 165^\circ$$

$$= -0.578$$

$$\sigma(3)_{Chebyshev} = 0.4254\,\omega_o \cos 135^\circ = -0.423$$

$$\sigma(2)_{Chebyshev} = 0.4254\,\omega_o \cos 105^\circ = -0.155$$

The values of the damping ratio for the respective roots, zeta, will be used on the parameter plane of Figure 10 because of their quality of maintaining the same value completely independent of any frequency scaling or frequency normalization that must now be performed in order to take advantage of the information contained on the parameter plane plot of Figure 10 which, it must be recalled, has been normalized to a "notch" frequency equal to 1.0.

To calculate the values for zeta the following relationship must be applied:

$$\zeta = \frac{|\sigma|}{\sqrt{\sigma^2 + \omega^2}}$$

Thus:

$$\zeta(1)_{Cheb} = \frac{.578}{\sqrt{0.334 + 0.132}} = 0.847$$

$$\zeta(3)_{Cheb} = \frac{0.423}{\sqrt{0.179 + 0.988}} = 0.392$$

$$\zeta(2)_{Cheb} = \frac{0.155}{\sqrt{0.024 + 1.843}} = 0.113$$

In summary, the following values will produce a Chebyshev configuration:

Circuit #1

$\zeta(1) = 0.847$

$\omega(1) = 0.364$

$\sigma(1) = -0.578$

Circuit #3

$\zeta(3) = 0.392$

$\omega(3) = 0.994$

$\sigma(3) = -0.423$

Circuit #2

$\zeta(2) = 0.113$

$\omega(2) = 1.358$

$\sigma(2) = -0.155$

Since the parameter plane of Figure 10 is normalized to a "notch" frequency of 1.0, the calculated frequencies listed above must be divided by the transfer function scaling factors, a, discussed in Section 2.2.

The scale factors for circuit #3 and circuit #2 were 1.234 and 1.566.

Therefore the parameter plane frequencies, ω_p, are:

$$\omega_p(3) = \frac{0.9941}{1.234} = 0.8056$$

$$\omega_p(2) = \frac{1.358}{1.566} = 0.867$$

To obtain these parameter plane values of omega and zeta, enter Figure 10 to determine the required values of normalized input capacitance, C, and amplifier gain, K. The following parameter values will result:

<u>Circuit #3</u>

K(3) = 1.605 a(3) = 1.234

C(3) = 0.100

<u>Circuit #2</u>

K(2) = 2.04 a(2) = 1.566

C(2) = 0.107

Refer to Appendix C for algebraic verification that a Chebyshev configuration has been formed by the parameter values selected above.

5.3. Evaluation of Response of System with Pole Configuration Derived Through the Parameter Plane.

What is the effectiveness of the parameter plane in realization of the "desired performance", which in this case was constant ripple in the pass band? Program SYS RESP, presented in Section 3 was used to conduct this evaluation.

The overall system transfer function required as input data to Program SYS RESP demands proper frequency scaling of the individual circuit transfer functions according to the formula derived in Section 2:

$$T_o(p) = \frac{K(p^2 + a^2)}{(3C + 1)p^2 + a\ (3.0 - 1.5K) + 3C\ p + a^2}$$

Circuit #3

$$T_{o3}(p) = \frac{1.605(p^2 + 1.522)}{1.30p^2 + 1.10p + 1.522}$$

Circuit #2

$$T_{o2}(p) = \frac{2.04(p^2 + 2.449)}{1.321p^2 + 0.408p + 2.449}$$

The fixed pole associated with circuit #1 retains its original circuit transfer function:

$$T_{o1}(p) = \frac{.9316}{p^2 + 1.1553p + .4658}$$

The results of the Program SYS RESP evaluation of the active filter system designed through the application of constant zeta and constant omega curves on the parameter plane are shown in Figure 16. These results indicate that the design objective of obtaining constant ripple performance has been attained.

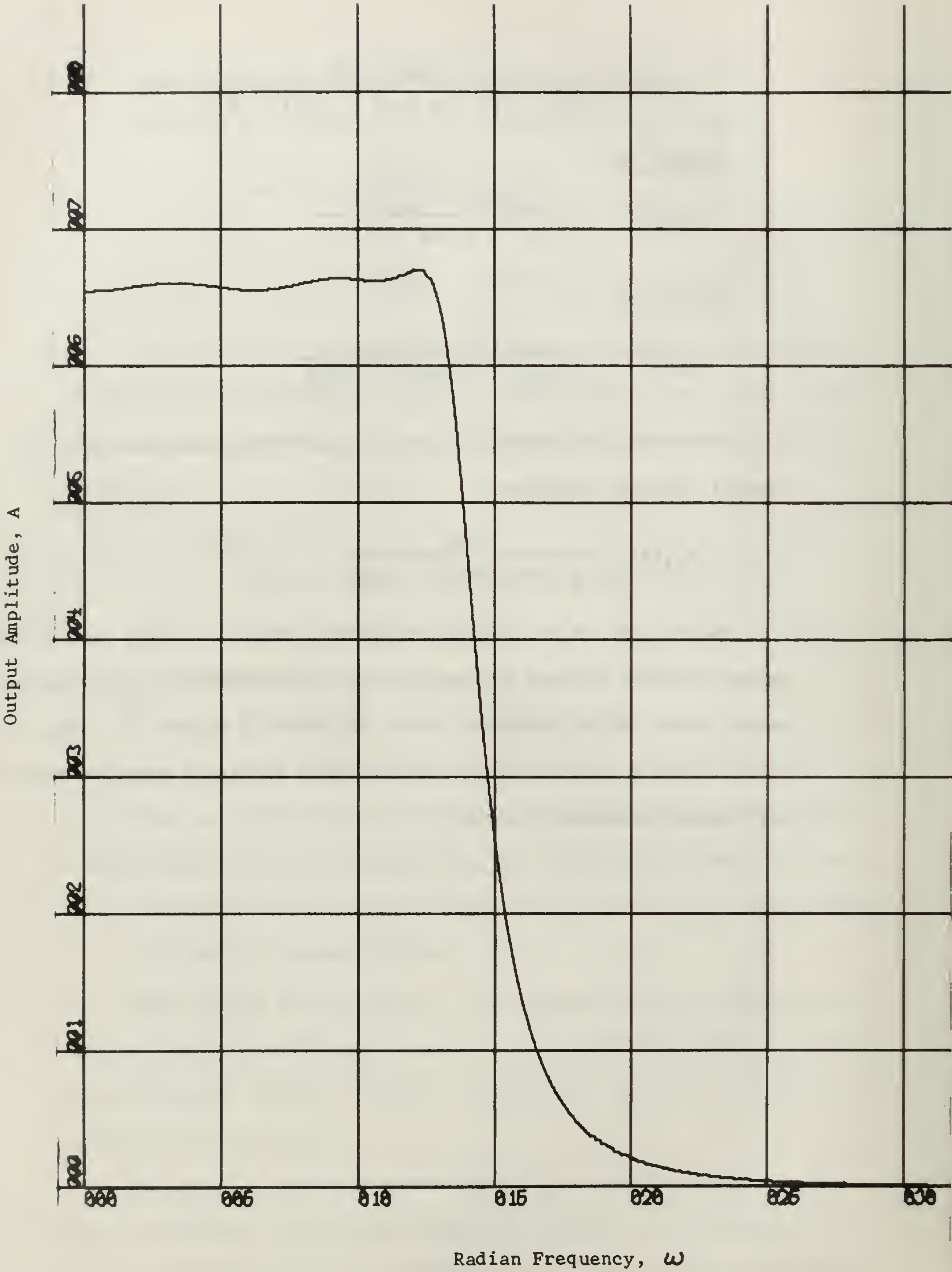

Figure 16. Digital Simulation of Chebyshev System Frequency Response

6. Conclusions

It has now been established that the constant bandwidth curves on the parameter plane and the constant zeta and omega curves on the parameter plane provide valid information which may be used in the design of active filter systems.

The constant bandwidth curves can provide a very accurate indication of filter performance which is directly related to parameter values. In addition the constant bandwidth curves provide the filter designer with a good qualitative indication of parameter values which should be avoided because of extreme sensitivity of filter performance to variations in those specific parameter values. Such regions of increased sensitivity are easily recognizeable by the increased density of the contour lines on the constant bandwidth plots.

When desired filter performance can be realized with a known pole configuration, the constant zeta and omega curves provide a very effective means of achieving that desired performance through the proper specification of the various parameter values. These curves can also provide the user with information pertinent to the tuning of the filter when performance is affected by tolerances of the circuit elements or the aging of components.

It should be noted, however, that the frequency normalized parameter plane can yield meaningful information only with proper frequency scaling. Considerable care must be exercised to correctly frequency scale the transfer functions when several circuits and/or systems are cascaded to achieve some desired result.

Subject to these conditions parameter plane techniques are useful tools in the design of active filter systems. The information contained on various parameter plane presentations should lead to much greater insight into the problem of correlating parameter values, pole locations and filter performance. Used in conjunction with the tremendous flexibility and freedom that prevails in the relocation of active filter poles, these parameter plane techniques should be substantial assets in further development of filter design theory.

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APPENDIX A

```
-COOP,,NAKAGAWA,0749/S/15/25/E/45=54,15,10000,4.
-FTN,E,L.
      PROGRAM FILTER
      DIMENSION XX(200) ,YY(200)
      DIMENSION ID(12),LA(32),A(32),WS(27),Y(200)
      DIMENSION XR1(200),XR2(200),ETAR(200)
      DIMENSION DUM1(5), DUM2(5)
      FA(AA,ZK,BW) = (       BW*BW*((ZK+1.)**2)/(ZK*ZK) ) -
     1              ((1.-2.* BW*BW+BW**4)/AA**2 )
      FB( ZK,BW)   = ( -4. * (ZK+1.)**2 * BW**2 )/(ZK*ZK)
      FC ( ZK,BW )  = ( -2. * (ZK+1.)**2 * BW**2 ) / ZK
      FD (ZK,BW )   = ( 2.*BW*BW)*( -(ZK+1.) + (ZK+1.)*BW*BW
     1              + 2.*(ZK+1.)**2 /ZK   )
      FE (ZK,BW )   = ((ZK+1.)**2 *BW*BW) * (BW*BW+1.)
      FF (ZK,BW )   = 1. - 2.*BW*BW + BW**4 +  4.*(ZK+1.)**2 *BW*BW/ZK**2
      READ 56,(ID(IX),IX=1,10)
   56 FORMAT( 10A8)
      I1=1
      I2=5
      ID(12)=8H(10**-2)
      LA(1)=8HA1
      LA(2)=8HA2
      LA(3)=8HA3
      LA(4)=8HA4
      LA(5)=8HA5
      LA(6)=8HA6
      LA(7)=8HA7
      LA(8)=8HA8
      LA(9)=8HA9
      LA(10)=8HA10
      LA(11)=8HA11
      LA(12)=8HA12
      LA(13)=8HA13
      LA(14)=8HA14
      LA(15)=8HA15
      LA(16)=8HA16
```

```
      LA(17)=8HA17
      LA(18)=8HA18
      LA(19)=8HA19
      LA(20)=8HA20
      LA(21)=8HA21
      LA(22)=8HA22
      LA(23)=8HA23
      LA(24)=8HA24
      LA(25)=8HA25
      LA(26)=8HA26
      LA(27)=8HA27
      LA(28)=8HA28
      LA(29)=8HA29
      LA(30)=8HA30
      LA(31)=8HA31
      LA(32)=8HA32
      DO 500 I=1,5
      DUM1(I)=0.0
  500 DUM2(I)=0.0
      LDUM=8H
C     FOR EACH VALUE OF OMEGA MUST PROVIDE A SET OF VALUES OF A, SET IS DEFINED
C    FIRST CARD   COL 9,10= MAX NUMBER OF A, (1,32),  COL 11,20 = VALUE OF OMEGA
C     2 TO 5 CARDS ,  8 VALUES OF A PER CARD USING 8F10.0 FORMAT
      KTPG=1
C     FOR EACH RUN SUBMIT WITH THE INDEX OF BEGINNING WB = 11  AND THE  INDEX OF
C        THE LAST WB = 12
      MAXY = 104
      DELY = 0.01
      Y(1)=0.
      DO 10 I=2,MAXY
   10 Y(I)=Y(I-1)+DELY
C        AK IS A VARIABLE SUPPLIED BY THE USER
      AK=2.0
      DO 100 I=11,12
      READ 60,MAXA,WB(I),(A(IX),IX=1,MAXA)
   60 FORMAT (I10,F10.0/(8F10.0))
```

```
      ID(7)=IBCD(MAXA,5,5,IER)
      ITMP=WD(I)*100.+.05
      ID(11)=IBCD(ITMP,5,5,IER)
      OMEGA=WD(I)
      COEFB=FB(XN,OMEGA)
      COEFC=FC(XN,OMEGA)
      COEFD=FD(XN,OMEGA)
      COEFE=FE(XN,OMEGA)
      COEFF=FF(XN,OMEGA)
      KOUNT=1
      DO 100 J=1,MAXA
      ARGA=A(J)
      DO 80 L=1,MAXY
   80 ISTAR(L)= 8H
      NEWPG=XMODF(J,4)
      IF(NEWPG-1)22 ,21,22
  199 PRINT 198
  198 FORMAT ( 11H ERROR STOP  )
      STOP
   21 PRINT 50,XN,KTPG
      KTPG = KTPG + 1
   50 FORMAT (    70H1          CONSTANT  BANDWIDTH  RELATIONS  ELLIPTIC
     1FILTER          K=        F5.1,10X,5HPAGE     ,I5)
   22 PRINT 51, OMEGA, ARGA
   51 FORMAT (20X,10H OMEGA(B)=     F5.2,10X, 5H A=   F5.5 )
      PRINT 52
   52 FORMAT (     117H     I          X1          X2         Y          X1
     1     X2          Y          X1          X2         Y          X1          X
     22                //)
      COEFA=FA(ARGA,XN,OMEGA)
      DO 99 L=1,MAXY
      CONST=COEFD*Y(L)+COEFE*Y(L)**2+COEFF
      BTERM=COEFB+COEFC*Y(L)
C     SOLVE QUADRATIC   COEFA X**2  +  BTERM X + CONST
      Z=BTERM**2-4.*COEFA*CONST
      IF(Z) 15,20,20
```

```
C     COMPLEX ROOTS  GO TO 19
   20 Z=SQRTF(Z)
      ANS1=(-BTERM+Z)/(2.*COLFA)
      ANS2=(-BTERM-Z)/(2.*COLFA)
      XN1(L)=ANS1
      XN2(L)=ANS2
   99 CONTINUE
      PRINT 33,(Y(II),XN1(II),XN2(II),ISTAR(II),II=1,MAX)
   33 FORMAT ( F5.2,2E11.2,A2,F5.2,2E11.2,A2, F5.2,2E11.2,A2,F5.2,2E11.
     12,A2  )
      PRINT 40
   40 FORMAT(/)
      DO 999 LL = 1,2
      GO TO (600,700),LL
  600 JL = 1
      DO 610 IL=1,MAXY
      IF (XN1(IL))   610,610,601
  601 XX(JL) = XN1(IL)
      YY(JL) = Y(IL)
      JL=JL+1
  610 CONTINUE
      MAX = JL - 1
      GO TO 800
  700 JL = 1
      DO 710 IL=1,MAXY
      IF(XN2(IL))  710,710,701
  701 XX(JL) = XN2(IL)
      YY(JL) = Y(IL)
      JL = JL + 1
  710 CONTINUE
      MAX = JL - 1
  800 IF (MAX-2)  200,200,300
  300 IF (KOUNT - 1) 301,302,301
  301 IF (J-MAXA)   320,303,320
  303 IF (LL-2) 320,310,310
  310 MODE=3
```

```
      GO TO 55
  502 MODE = 1
      KOUNT=KOUNT + 1
      GO TO 55
  520 MODE=2
      GO TO 55
  200 IF(J-MAXA) 201,400,201
  201 GO TO 999
  400 CALL DRAW (3,DUM1,DUM2,3,0,LDUM ,ID,.5,.1,0,0,0,0,9,15,0,LAST)
      GO TO 100
   55 CALL DRAW(MAX, XX,YY,MODE,0,LA(J),ID,.5,.1,0,0,0,0,9,11,1,LAST)
  999 CONTINUE
  100 CONTINUE
      STOP
   19 ISTAR(L) = 8H*
      XK1(L)= XK2(L)=0.
      GO TO 99
         END
```

```
JOB 0600 G.R.NAKAGAWA  ELLIPTIC FILTER   K = 2.0        VALUES OF A,    OMEGA =
      16     1.30
.1585     .1778     .1995     .2239     .2512     .2818     .3162     .3548
.3981     .4467     .5012     .5623     .6310     .7079     .7943     .8913
      16     1.40
.3981     .4467     .5012     .5623     .6310     .7079     .7943     .8913
1.000     1.122     1.259     1.413     1.585     1.778     1.995     2.239
      16     1.60
.3981     .4467     .5012     .5623     .6310     .7079     .7943     .8913
1.000     1.122     1.259     1.413     1.585     1.778     1.995     2.239
      16     1.80
.3981     .4467     .5012     .5623     .6310     .7079     .7943     .8913
1.000     1.122     1.259     1.413     1.585     1.778     1.995     2.239
```

APPENDIX B

```
..JOB0600F, NAKAGAWA,G.R.
      PROGRAM FREQRESP
C     THIS PROGRAM WAS DEVELOPED TO EVALUATE THE FREQUENCY
C     RESPONSE OF THE PARALLEL-T ACTIVE FILTER WITH A TRANSFER
C     FUNCTION HAVING A NUMERATOR POLYNOMIAL OF THE FORM
C
C              K * (P**2 + 1)
C
C     AND A DENOMINATOR POLYNOMIAL OF THE FORM
C
C        ((SK+1)*C+1)*P**2 + ((SK+1)*(2-K)/SK+(SK+1)*C)*P + 1
C
C     WHERE
C        SK IS THE RATIO OF ELEMENT VALUES
C        C  IS THE NORMALIZED CAPACITANCE
C
C     EACH PLOT WILL BE FOR A FIXED VALUE OF SK AND C, BUT
C     A FAMILY OF CURVES WILL BE PLOTTED TO REPRESENT VARIOUS
C     VALUES OF AMPLIFIER GAIN K.
C
      DIMENSION W(500),LA(40),XK(40),ITITLE(12),A(5),B(5),
     1BREAL(500),BIMAG(500),BMAGN(500),AREAL(500),AIMAG(500),
     2DB(500), AMAGN(500)
      READ 200, (ITITLE(I),I=1,6)
      READ 200, (ITITLE(I), I=7,12)
      LA(1) = 4HK1.6
      LA(2) = 4HK1.8
      LA(3) = 4HK2.0
      LA(4) = 4HK2.2
      LA(5) = 4HK2.4
      LA(6) = 4HK2.6
      LA(7) = 4HK2.8
      LA(8) = 4HK3.0
      LA(9) = 4HK3.2
      LA(10) = 4HK3.4
      LA(11) = 4HK3.6
      LA(12) = 4HK3.8
      LA(13) = 4HK4.0
      LA(14) = 4HK4.2
      LA(15) = 4HK4.4
      LA(16) = 4HK4.6
      SK = 6.0
      C = 0.60
      MAXK = 16
      GAIN = 1.4
      W(1) = 0.000
      NUMPTS = 200
      DO 100 I = 1, MAXK
      GAIN = GAIN + 0.2
```

```
      XK(I) = GAIN
      B(1) = 1.0
      B(3) = 1.0
      A(1) = 1.0
      A(2) = (2.0-XK(I))*(SK+1.0)/SK + (SK+1.0)*C
      A(3) = 1.0 + (SK+1.0)*C
      DO 25 J = 2,200
      W(J) = W(J-1) + 0.01
      BREAL(J)  = B(1) - B(3)*W(J)**2
      BIMAG(J) = B(2)*W(J)
      BMAGN(J) = SQRTF(BREAL(J)**2 + BIMAG(J)**2)
      AREAL(J) = A(1) - A(3)*W(J)**2
      AIMAG(J) = A(2)*W(J) - A(4)*W(J)**3
      AMAGN(J) = SQRTF(AREAL(J)**2 + AIMAG(J)**2)
      TEMP = XK(I)*BMAGN(J)/AMAGN(J)
      DB(J) = 8.68589* LOGF(TEMP)
   25 CONTINUE
      IF (I- 1)   33,33,35
   33 MODE = 1
      GO TO 55
   35 IF (MAXK - I) 37,37,36
   36 MODE = 2
      GO TO 55
   37 MODE = 3
      GO TO 55
   55 CALL DRAW(NUMPTS,W,DB,MODE,0,LA(I),ITITLE,.2 ,5.,8,
     10,2,0,9,15,1,LAST)
  100 CONTINUE
  200 FORMAT (6A8)
      END
      END

JOB 0600 NAKAGAWA   PARALLEL-T ACTIVE FILTER
X=W(J)     Y=DB OUT    SK=6.0     C=0.60
```

APPENDIX C

Algebraic Verification of Chebyshev Pole Configuration

The parameter plane techniques of Section 5 should yield parameter values which will result in the desired Chebyshev pole configuration. An algebraic evaluation of the characteristic equations for circuits 1, 2, and 3 will now be conducted to confirm that such a Chebyshev pole configuration has, in fact, been attained.

The parameter plane produced the following frequency scaled transfer functions for circuits 1, 2 and 3 respectively:

$$T_{o1}(p) = \frac{0.9316}{p^2 + 1.1553p + 0.4658}$$

$$T_{o2}(p) = \frac{2.04(p^2 + 2.449)}{1.321p^2 + 0.408 + 2.449}$$

$$T_{o3}(p) = \frac{1.605(p^2 + 1.522)}{1.30p^2 + 1.100p + 1.522}$$

The characteristic equations (denominator polynomials) for these transfer functions are of the form

$$Ap^2 + Bp + C = 0$$

$$p^2 + \frac{B}{A}p + \frac{C}{A} = 0$$

Thus the natural frequency, ω_n, for the equation is:

$$\omega_n = \sqrt{\frac{C}{A}}$$

since

$$2\zeta\omega_n = \frac{B}{A}$$

$$\zeta = \frac{B}{2A\omega_n} = \frac{B\omega_n}{2C}$$

and the imaginary component of the pole becomes

$$\omega = \omega_n\sqrt{1-\zeta^2}$$

Circuit 1

$$\omega_n(1) = 0.4658 = 0.6825$$

$$\zeta(1) = \frac{(1.155)(0.6825)}{2(0.4658)} = 0.847$$

$$\omega(1) = (0.6825)(0.5331) = 0.364$$

Circuit 2

$$\omega_n(2) = \frac{2.449}{1.321} = 1.362$$

$$\zeta(2) = \frac{(0.408)(1.362)}{2(2.449)} = 0.1132$$

$$\omega(2) = (1.362)(0.9936) = 1.354$$

Circuit 3

$$\omega_n(3) = \frac{1.522}{1.30} = 1.082$$

$$\zeta(3) = \frac{(1.100)(1.082)}{2(1.522)} = 0.391$$

$$\omega(3) = (1.082)(0.9204) = 0.996$$

Comparison of these calculated values of zeta and omega with the desired Chebyshev values of Section 5.2 substantiates the effectiveness of the constant zeta and omega curves on the parameter plane for the design task of obtaining a desired active filter system pole configuration.

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1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

PARAMETER PLANE TECHNIQUES FOR ACTIVE FILTER SYSTEMS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Thesis, M.S., December 1966

5. AUTHOR(S) *(Last name, first name, initial)*

NAKAGAWA, Gordon R.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1966	82	6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
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13. ABSTRACT

Since the introduction of parameter plane techniques by Siljak in 1964, much effort has been devoted to the application of these techniques to various control system problems.

This paper concerns the application of parameter plane techniques to the design of active filter systems.

Constant bandwidth curves on the parameter plane are compared with frequency response curves to validate the constant bandwidth curves as accurate representations of active filter performance. This in turn implies that the constant bandwidth curves can be used in the design of active filter systems.

Also presented is an example of the use of constant zeta and omega curves on the parameter plane to manipulate the pole locations of individual circuits to achieve a desired overall system configuration with the proper frequency scaling.

DD FORM 1 JAN 64 1473

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